Connecting Measurements in Surveying and its Problems

Michaela KORBAŠOVÁ, Slovak Republic and Jaroslav MAREK, Czech Republic

Key words: connecting measurements, two-stage regression model, BLUE, H-optimum estimator.

SUMMARY

Even if geodetic networks were built and are still building with maximal possible accuracy nevertheless they reflect knowledge level or technical and instrumental equipment of its age. Development of practical and theoretical knowledge as well as development of measuring and PC technique leads step by step to difference between possibilities of existing network and its state.

Connecting measurements in surveying are almost by each task. Let's have a look at the local geodetic networks with the higher accuracy, which create a foundation for all constructional and spatial demanding building structures (i.e. nuclear power stations, bridges, water works, etc.). These are indeed adjusted in an independent homogenous block in a local coordinate system but at the end it is connected to the State trigonometric network for purpose of documentation and mapping. Another situation rise when a density of existing network isn't sufficient and it is necessary to make more dense by measuring in the next stages. In this case we connect network by measuring in the next stage (connecting network) to the existing one (connected network). As a mathematical mechanism it is usually used Least Square Method (LSM). Modernization and development of geodetic foundations in Slovakia although in present days is in progress, a presentation of idea of new solution of connecting measurements and H-optimum estimator will always have its importance. Within research activities we suggest and test new algorithms for processing of various connecting measurements resulting from geodetic practice.

It is important to note that the new way of solution of connecting measurements is important almost always, when we are connected to the network with lower order accuracy than accuracy of connecting network. Another example is to determine only a part of connecting structure in a certain area of interest with higher accuracy and at the same time unbiased estimators have to fulfil constrains on parameters of both stages, which has to be adhered by parameter estimation of the second stage. In this case LSM isn't sufficient. We suggest an optimal mathematical model of geodetic measurement in two stages with respecting of constraints putting on model parameters. New knowledge in mathematical statistics (creation of new optimality type, H-optimality) creates a possibility to investigate geodetic measurements from the new point of view. Mathematical mechanism introduced from the above mentioned results will be able to use not only in Slovakia and Czech Republic but also in other countries where it will be necessary to solve similar problems of connecting measurements.

Connecting Measurements in Surveying and its Problems

Michaela KORBAŠOVÁ, Slovak Republic and Jaroslav MAREK, Czech Republic

1. INTRODUCTION

We come into contact in geodesy with connecting measurements in various forms very frequently therefore we think that problem we are dealing with in this paper, is actual not only in our country but equally also in other countries. We introduce a mathematical model of geodetic measurement in two stages, by which we solve how to minimize the influence of parameter uncertainties of the first stage on the selected function of parameters of the second stage. Because it is about processing (optimization) of realised measurement, which realisation we can't influence any more, in comparison of classic type of optimization (Kubáčková, 1990), the main problem, which we have to be solved is concerning with existence of non-neglected errors by parameter estimation of connecting network. But in this case there are no jointly effective estimators of parameters of connected network and therefore it is necessary to load a new type of estimator optimization (H-optimum estimator as $\tilde{\beta}$), which is not identical with given optimization of least square method (LSM estimator as $\hat{\beta}$). Recommendation, which of these two processes is preferable, will be admit on a base of variations between estimated parameters of LSM and H-optimum estimator and on a base of complementation of mathematical interpretation with geodetic one.

2. MATHEMATIC THEORY AND FORMULATION OF PROBLEM

Base type of model, which is a point of research, is "two stage model of indirect measurement with constraints of type I and II". For understanding of terms as are constraints of type I and II see (Kubáček, 1993, 2004). Mathematically it is possible this model express as follows

$$\begin{pmatrix} \mathbf{Y}_1 \\ \mathbf{Y}_2 \end{pmatrix} \approx \left[\begin{pmatrix} \mathbf{X}_1 & \mathbf{0} \\ \mathbf{D} & \mathbf{X}_2 \end{pmatrix} \begin{pmatrix} \mathbf{\Theta} \\ \boldsymbol{\beta} \end{pmatrix}, \begin{pmatrix} \mathbf{\Sigma}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{\Sigma}_2 \end{pmatrix} \right], \tag{1}$$

where $\mathbf{Y}$ is random vector of model of connecting measurement expressed as $Y = (\mathbf{Y}_1, Y_2)'$, X_1 is known matrix of type $n_1 x k_1$ (design matrix in the first stage), $\mathbf{X}_2$ is known matrix of type $n_2 x k_2$ (design matrix in the second stage), D is known matrix of type $n_2 x k_1$, which realises interconnection between the first and second stage, n_1 is number of measured quantities in the first stage, n_2 is number of measured quantities in the second stage, k_1 is number of unknown quantities in the first stage, k_2 is number of unknown quantities in the second stage, Θ is unknown k_1 – dimensional parameter, which is estimated on a base of vector Y_1 in the first stage, β is unknown k_2 – dimensional parameter, which is estimated on a base of vectors $(\mathbf{Y}_2 - \mathbf{D}\hat{\mathbf{\Theta}})$ and $\hat{\mathbf{\Theta}}$, Σ_1 is covariance matrix of the first stage, Σ_2 is covariance matrix of the second stage. For constraints of type I have to be fulfilled

$$\mathbf{a} + \mathbf{C}\hat{\mathbf{\Theta}} + \mathbf{B}\tilde{\boldsymbol{\beta}} = \mathbf{0}, \tag{2}$$

where B is matrix of partial derivations of function formulas of constraints for parameter β, C is matrix of partial derivations of function formulas of constraints for parameter Θ, a is vector of constraints, $\tilde{\boldsymbol{\beta}}$ is estimator from the second stage, $\hat{\boldsymbol{\Theta}}$ is estimator from the first stage.

Model with constraint of type I as well as complete mathematical formulations of above mentioned notes are described in (Marek, 2003) or (Korbašová, 2003). Among $\tilde{\beta}$ estimators there is no jointly efficient estimator, it isn't possible to use LSM and therefore it is minimized at least variance of estimator of some quantity of function of β parameter. Estimator $\tilde{\beta}$ we called H-optimal, when we minimize function

$$\mathrm{f}(\tilde{\boldsymbol{\beta}}) = Tr[\mathbf{HVar}(\tilde{\boldsymbol{\beta}})],\ \tilde{\boldsymbol{\beta}} \in \tilde{U}_\beta, \qquad (3)$$

where H is given $k_2 x k_2$ positive semi-definite matrix, $\tilde{\mathrm{U}}_\beta$ is class of all linear unbiased estimators $\tilde{\beta}$ of β parameter in model (1) on a base of vectors $\mathbf{Y}_2 - \mathbf{D}\hat{\boldsymbol{\Theta}}$ and $\hat{\boldsymbol{\Theta}}$, which at the same time fulfil a constraint (2).

Accordingly it is possible to express linear unbiased jointly effective estimator of parameters β and γ in regular model with constraints of type II. Further particulars about this problem you can find in (Marek, 2004) or (Korbašová, 2003).

3. NUMERICAL STUDY

In this chapter we always offer results of classical process of estimation as LSM estimator ($\hat{\beta}$ estimator) and as H-optimum estimator ($\tilde{\beta}$ estimator) namely for two variants.

Variant SP (Similar Precision) assumes coordinate accuracy of points of existing network approximately the same as coordinate accuracy of connected points (m_{xy} is to 10 mm).

Variant VP (Various Precision) assumes coordinate accuracy of points of existing network app. 6 cm and coordinate accuracy of connected points to 10 mm (accuracy of measured parameters in the second stage assumes for angles 5 ^{cc} and for distances from 3 to 5 mm). By this we want to show that whether is accuracy in both stages similar or in the second stage is accuracy much more higher, by application of H-optimum estimator and by using of minimize criterion of $Tr[\mathbf{HVar}(\tilde{\boldsymbol{\beta}})]$, from the mathematical point of view we obtain always better results in accuracy of estimated parameter (see tab.1 to tab.6). Also we want to show that if connected structure is determined with the same accuracy as existing network, on which it is necessary to connect realised measurement, variances obtained by H-optimum estimator are only negligible better and therefore in this case is LSM sufficient (see in examples tab.1, tab.3, tab.5).

As it will be possible to see from tables of results for H-optimum estimators by *variant VP*, accuracy improvement of one quantity is on the expense of accuracy of other elements of H-optimum estimators and therefore structure of matrix H will be important (in tab.2, 4, 6 see columns of H-optimum estimators). It is not possible to process it mechanically and it is necessary to modify matrix H according to requirements on result accuracy of obtained estimators. We want to notice at necessity of choice of this matrix according to situation. Here is opened a possibility to eliminate experimentation with choice of matrix H (for purpose of substitution of given aim by user) with help of mathematical process of looking for minimum of functional on a set of admissible solutions. This modification of matrix H assumes our next cooperation, in which it will be developed computational algorithm for determination of optimal structure of matrix H according to mentioned requirements.

3.1 Example 1

Let us imagine the following situation. Let's have points F1, F2 and F3 of existing network and points P1 and P2, for which it is necessary to determine optimal estimators of coordinates (Fig.1). Available are coordinate estimators of points of existing network (F2 (Θ_3, Θ_4), F3 (Θ_5, Θ_6)) from the first stage of measurement. In the second stage are measured three distances (β1, β2, β3) and two angles (β4, β5).

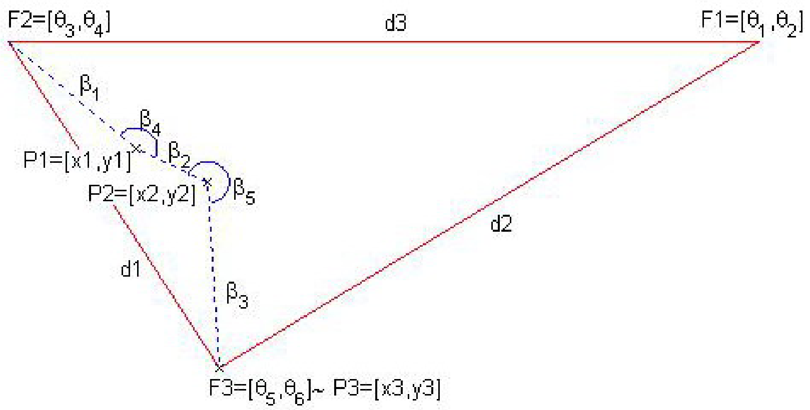

Fig.1: Presentation of situation for Example 1

We determined a standard LSM estimator and H-optimum estimator of measured parameters and their accuracy characteristics for both mentioned variants (*variant SP* and *variant VP*). Furthermore within *variant VP* we determined H-optimum estimators for various structures of matrix H (it depends on which quantity we would like to determine with higher accuracy). Because it is valid that $Tr(HVar(\hat{\beta})) > Tr(HVar(\tilde{\beta}))$, we can say that from mathematical point of view by application of H-optimum estimator we obtained better accuracy results than LSM estimate. However significant improvement of accuracy is in case of *variant VP*. Besides we can see that accuracy improvement of one vector element of H-optimum estimators is on the expense of accuracy of other quantities.

Tab. 1: variant SP

LSM estimator	Variance for LSM estimator	H-optimal estimator	Variance for H-optimal estimator
216.352 m	4.4 mm	216.350 m	4.4 mm
103.099 m	4.6 mm	103.098 m	4.6 mm
245.482 m	4.5 mm	245.481 m	4.5 mm
183.1303 g	4.9 ^{cc}	183.1294 g	7.9 ^{cc}
267.8715 g	4.9 ^{cc}	267.8700 g	12.6 ^{cc}
$Tr(HVar(\hat{\beta}))=0.000061 > Tr(HVar(\tilde{\beta}))=0.000058$			

Tab. 2: variant VP

LSM estimator	Variance for LSM estimator	H-optimal estimator (1)	Variance for H-opt. estimator (1)	H-optimal estimator (2)	Variance for H-opt. estimator (2)
216.352 m	44.6 mm	216.347 m	5.0 mm	216.351 m	40.2 mm
103.099 m	39.5 mm	103.095 m	5.0 mm	103.099 m	35.6 mm
245.482 m	40.9 mm	245.478 m	5.0 mm	245.482 m	36.9 mm
183.1303 g	6.8 ^{cc}	183.1275 g	253.0 ^{cc}	183.1298 g	48.4 ^{cc}
267.8715 g	10.0 ^{cc}	267.8664 g	472.4 ^{cc}	267.8711 g	45.2 ^{cc}
$Tr(HVar(\hat{\beta}))=0.005220 > Tr(HVar(\tilde{\beta}))=0.000076$ ....for **(1)** $Tr(HVar(\hat{\beta}))=0.005241 > Tr(HVar(\tilde{\beta}))=0.004794$ ....for **(2)**					

3.2 Example 2

The second example documents situation where in comparison with Example 1 two more quantities were measured in connected structure (Fig.2). The coordinate estimators of points of existing network (F1 (Θ_1, Θ_2), F2 (Θ_3, Θ_4), F3 (Θ_5, Θ_6)) from the first stage of measurement are available. Three distances (β5, β6, β7) and four angles (β1, β2, β3, β4) are measured in the second stage. Measurement accuracy is characterized by particular covariance matrixes. Again there were determined LSM estimators and also H-optimum estimators for *variant SP* and *variant VP*.

Equally in this example within *variant VP* we determined H-optimum estimators for various structures of matrix *H* (it depends on which quantity we would like to determine with higher accuracy). Because it is valid that $Tr(HVar(\hat{\beta})) > Tr(HVar(\tilde{\beta}))$, we can say that from mathematical point of view by using of H-optimum estimator we obtained better accuracy results than LSM estimator. However significant improvement of accuracy is in case of *variant VP*. Besides we can see that accuracy improvement of one vector element of H-optimum estimators is on the expense of accuracy of other quantities. Results are presented in tab.3 and tab.4.

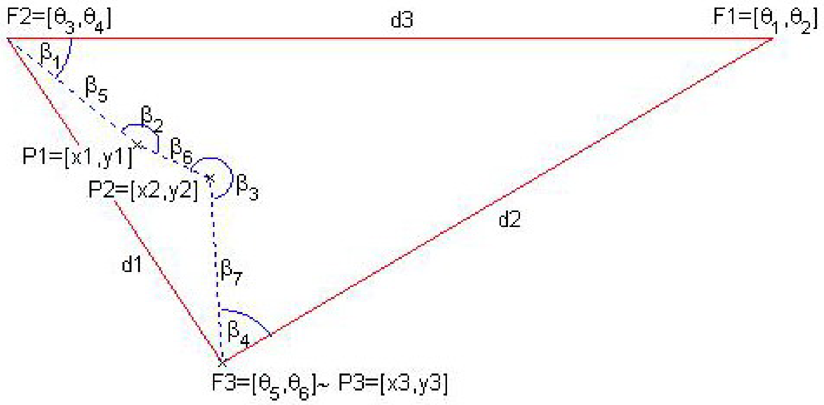

Fig. 2: Presentation of situation for Example 2

Tab. 3: variant SP

LSM estimator	Variance for LSM estimator	H-optimal estimator	Variance for H-optimal estimator
45.01130 g	4.5 ^{cc}	45.01181 g	7.1 ^{cc}
183.13038 g	4.6 ^{cc}	183.13048 g	4.6 ^{cc}
267.87156 g	4.6 ^{cc}	267.87146 g	4.9 ^{cc}
68.78246 g	4.5 ^{cc}	68.78217 g	6.1 ^{cc}
216.3497 m	4.8 mm	216.3489 m	4.7 mm
103.0958 m	4.8 mm	103.0959 m	4.8 mm
245.4847 m	4.8 mm	245.4819 m	3.9 mm
$Tr(HVar(\hat{\beta}))=0.000068 > Tr(HVar(\tilde{\beta}))=0.000060$			

Tab. 4: variant VP

LSM estimator	Variance for LSM estimator	H-optimal estimator (1)	Variance for H-opt. estimator (1)	H-optimal estimator (2)	Variance for H-opt. estimator (2)
45.0113 g	23.6 ^{cc}	45.0126 g	179.4 ^{cc}	45.0114 g	48.2^{cc}
183.1304 g	17.5 ^{cc}	183.1307 g	25.2 ^{cc}	183.1305 g	49.2 ^{cc}
267.8716 g	31.8 ^{cc}	267.8714 g	60.7 ^{cc}	267.8714 g	48.9 ^{cc}
68.7825 g	52.1 ^{cc}	68.7818 g	139.5 ^{cc}	68.7824 g	45.5 ^{cc}
216.350 m	24.2 mm	216.347 m	5.0 mm	216.351 m	43.7 mm
103.096 m	9.7 mm	103.095 m	5.0 mm	103.097 m	10.1 mm
245.485 m	87.8. mm	245.478 m	5.0 mm	245.483 m	69.4 mm
$Tr(HVar(\hat{\beta}))=0.008392 > Tr(HVar(\tilde{\beta}))=0.000075$ ....for **(1)** $Tr(HVar(\hat{\beta}))=0.004199 > Tr(HVar(\tilde{\beta}))=0.003722$ ....for **(2)**					

3.3 Example 3

Because we would like to investigate how can various shape of connected structure influence accuracy of parameter estimators, we created a modification of Example 1 with three connected points (Fig.3). Four distances (β1, β2, β3, β4) and three angles (β5, β6, β7) are measured. Again there were determined LSM as well as H-optimum estimators for mentioned variants. Results are presented in tab.5 and tab.6.

Tab. 5: variant SP

LSM estimator	Variance for LSM estimator	H-optimal estimator	Variance for H-optimal estimator
162.254 m	4.5 mm	162.255 m	4.4 mm
77.991 m	4.7 mm	77.992 m	4.6 mm
184.014 m	4.6 mm	184.016 m	4.6 mm
132.680 m	4.5 mm	132.681 m	4.4 mm
167.3478 g	5.0^{cc}	167.3490 g	7.5^{cc}
276.4303 g	5.0^{cc}	276.4342 g	18.7^{cc}
150.8097 g	5.0^{cc}	150.8080 g	9.1^{cc}
$Tr(HVar(\hat{\beta}))=0.000083 > Tr(HVar(\tilde{\beta}))=0.000081$			

Tab. 6: variant VP

LSM estimator	Variance for LSM estimator	H-optimal estimator (1)	Variance for H-opt. estimator (1)	H-optimal estimator (2)	Variance for H-opt. estimator (2)
162.255 m	34.2 mm	162.260 m	5.0 mm	162.255 m	28.6 mm
77.991 m	27.7 mm	77.996 m	5.0 mm	77.991 m	28.8 mm
184.014 m	29.4 mm	184.020 m	5.0 mm	184.014 m	29.9 mm
132.680 m	33.7 mm	132.686 m	5.0 mm	132.680 m	29.3 mm
167.3478 g	5.1^{cc}	167.3529 g	259.0^{cc}	167.3488 g	49.3^{cc}
276.4303 g	6.2^{cc}	276.4466 g	825.8^{cc}	276.4312 g	49.7^{cc}
150.8097 g	5.2^{cc}	150.8028 g	345.9^{cc}	150.8087 g	49.6^{cc}
$Tr(HVar(\hat{\beta}))=0.003940 > Tr(HVar(\tilde{\beta}))=0.000102$ ....for **(1)** $Tr(HVar(\hat{\beta}))=0.179424 > Tr(HVar(\tilde{\beta}))=0.165153$ ....for **(2)**					

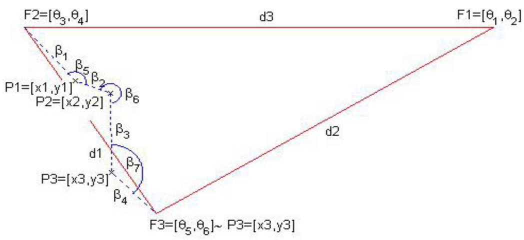

Fig.3: Presentation of situation for Example 3

4. CONCLUSION

New knowledge in mathematical statistics (creation of new type of optimization, so called H-optimality) creates a space for investigation of geodetic measurements from the new point of view. The way of solution of connecting measurements with suggested new process is important when we connect to the network, which accuracy is much lower as coordinate accuracy of points of connected network. Another exploitation of H-optimality rises when we are in such situation in geodetic practice where there is a requirement to determine only a part of connected structure in particular area of interest with higher accuracy and furthermore unbiased estimators have to fulfil a constraint given on parameters of both stages. In this case LSM is not sufficient. Meanwhile it is not possible to introduce universal process of solution by H-optimum estimator, even though certain sequence of individual steps of calculation will be possible to realise according to our introduced algorithm. This problem demands to calculate a huge quantum of examples, various situations, which can rise in practice and in spite of it we think that geodetic practice is so miscellaneous that it will be always necessary to consider each individual arose situation and approach to it by individual certain way.

REFERENCES

Korbašová, M. 2003. Statistical Problems of Connecting Measurements in Surveying. Written Part of Dissertation Work. 36 p., Bratislava.

Kubáček, L. 1993. Two-stage Regression Models with Constraints. Math Slovaca 43, p. 643-658, Bratislava, Veda.

Kubáček, L. – Marek, J. 2004. Partial Optimum Estimator in Two-stage Regression Model with Constraints and a Problem of Equivalence. Math Slovaca 54, Bratislava, Veda (to appear).

Kubáčková, L. 1990. Methods of Experimental Data Processing. 325 p., Bratislava, Veda.

Marek, J. 2003. Estimation in Connecting Measurements. Acta Universitatis Palackianae Olomucensis, Facultas rerum naturalium, Mathematica 42 (2003), pp. 69-86, Olomouc, Palacky University Olomouc.

Marek, J. 2004. Estimation in Connecting Measurements with Constraints of Type II. Acta Universitatis Palackianae Olomucensis, Facultas rerum naturalium, Mathematica 43 (2004), (to appear), Olomouc, Palacky University Olomouc.

BIOGRAPHICAL NOTES

Korbašová Michaela, MSc.
Study Geodesy and Cartography SUT Bratislava 1996-2001. Internal form of doctoral study at STU Bratislava from 2001-2002; continue external form of doctoral study from 2002. Lecturer at the Department of Surveying at SUT from 2002. Takes part in solution of grant project VEGA 1/0318/03. Author of several publications in various journals and conference proceedings and active participation at several conferences.

Marek Jaroslav, Mgr.
Study Applied Mathematics at Palacky University Olomouc, 1995. Doctoral study from 2002. Lecturer at the Department of Mathematical Analysis and Applied Mathematics, Faculty of Science at Palacky University Olomouc from 1997. Author of several publications in various journals and conference proceedings and active participation at several conferences.

CONTACTS

Michaela Korbašová
Slovak University of Technology, Faculty of Civil Engineering, Department of Surveying
Radlinského 11
813 68 Bratislava
SLOVAK REPUBLIC
Tel. + 421 2 59274 391
Fax + 421 2 52494 334
Email: michaela.korbasova@stuba.sk
Web site: http://www.svf.stuba.sk/kat/GDE

Jaroslav Marek
Palacky University, Faculty of Science, Department of Mathematical Analysis and Applied Mathematics
Tomkova 40
779 00 Olomouc
CZECH REPUBLIC
Tel. + 420 585 634 606
Fax + 420 585 225 737
Email: Marek@inf.upol.cz
Web site: http://www.mant.upol.cz