The Algorithm of Urban Estates Valuation*

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SUMMARY

The urban estates are particular components of ground property. The urban estates are characterized by the ground components of great complication. Urbanized estates may be of following type: urban grounds, commercial premises, office buildings, welfare buildings, industrial buildings, warehouse buildings and small buildings of some type.

Transactions including urban estates are not very common at the real estates' market. However, if a transaction takes place, all components of the estate have an influence on the transaction price. The application of such transaction prices for the valuation of similar estates is very complicated, for estates' components structure is highly differentiated and it is not possible to establish estates' representative reference unit.

The algorithm enabling the sold industrial estates unit element's and transaction prices comparative analysis with elements of valuated urban estates will be presented in this paper. This algorithm is based on statistical modelling. Its result is the industrial estates' market value estimation.

The transaction price of every urban estate is considered as the function of market indices, its element's usable area and weight attributes indices and values. The most probable values of market indices and weight indices are estimated applying the least squares technique with application of Gauss-Markov model.

The market value calculation of urban estate is carried out by adding products of estimated price indicators, usable area of valuated estate and weight indices with its attributes estimated products.

All deliberations are carried out with full analysis of variance and interval estimation of estimated estate's market value application. Finally, the numerical verification of elaborated algorithm will be produced, on the example of urban estate situated in Cracow.

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1. INTRODUCTION

Urban real estates are the particular components of ground property. They are characterized by the ground components great complication. The description of urban real estates should include the following geometrical parameters: urban estates area, buildings of all types' usable areas, rooms of industrial use and warehouses areas and capacities, umbrella roof's usable areas.

Apart from geometrical description, some characteristic parameters of urban estates that may have significant influence on their market value should be taken into consideration. They are localization, building vicinity, building technical state and urban estate way of exploitation. The most important attribute for sold urban estate is its transaction prize. This prize includes prices of all urban estate's components.

There is no possibility to define fragmentary prices for urban estates components during the transaction process and preparation of authenticated deed. It requires the prior defining of proportion coefficients between their market values. These coefficients may be calculated with advanced statistical analysis for urban estates' market.

If the number of transactions (transaction prices) is higher than the number of price coefficients for each building and other urban estates installations and their attributes accepted for market analysis and estates valuation, than weight and price coefficients estimation will be based on Gauss-Markov parametric model.

If we define the following symbols as
C_{Tj} – transaction price for whole j-estate,
S_i – the area of every i-element (parcel, parcel parts having defined soil classes, flat or building usable areas or whole building),
c_i – i-element unit price,
$a_1, a_2, ...a_j$ -attributes accepted for urban estates estimation values,
$k_1, k_2, ..., k_j$ - the weight coefficients of attributes accepted for urban estates valuation,
then the parametric model for unit prices and attributes will get the following formula

$$C_{Tj} = S_1 \times c_1 + S_2 \times c_2 + ... + S_i \times c_i + a_1 \times k_1 + ... + a_s \times k_s . \qquad (1)$$

The estimated parameters in this formula are the unit prices coefficients (c_i) of estate's element and the weight coefficients (k_s) of elaborated attributes.

2. VALUATION MODEL PARAMETERS' ESTIMATION

The set of equations (1) written as the matrix, for similar estates transaction prices takes the following form:

$$\begin{bmatrix} C_{T1} \\ C_{T2} \\ \vdots \\ C_{Tj} \end{bmatrix} = \left\{ \begin{bmatrix} S_{11} & S_{12} & \dots & S_{1i} \\ S_{21} & S_{22} & \dots & S_{2i} \\ \vdots & \vdots & \dots & \vdots \\ S_{j1} & S_{j2} & \dots & S_{ji} \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1s} \\ a_{21} & a_{22} & \dots & a_{2s} \\ .. & .. & .. & .. \\ a_{j1} & a_{j2} & \dots & a_{js} \end{bmatrix} \right\} \times \left\{ \begin{bmatrix} c_1 \\ c_2 \\ . \\ c_i \end{bmatrix} \right\} + \begin{bmatrix} \delta_{T1} \\ \delta_{T2} \\ . \\ \delta_{Tj} \end{bmatrix} \quad (2)$$

where

- indices 1÷i point out elements of similar estates' elaborated group,
- indices 1÷s point out similar estates' elaborated attribute
- indices 1÷j point out the following number of similar estate's sell transaction number.

The set of equation above has solution in real numbers if the transaction number (j) is higher than number (i + s), that describes number of estate's elaborated elements and attributes.

After using block matrices notation, equation set (2) has the following form:

$$\{C_T\} = \{[S] \quad [a]\} \cdot \left\{ \begin{bmatrix} [c] \\ [k] \end{bmatrix} \right\} + \{\delta_T\}, \quad (3)$$

where:

$\{C_T\}$ - transaction prices vector, for whole estates,

$[S]$ - rectangular vertical matrix, containing areas of elements for elaborated estates

$[a]$ - rectangular vertical matrix, containing attributes of elaborated urban estates

$\{c\}$ -unit prices coefficients vector for estates' elements,

$[k]$ -weight coefficients vector for elaborated attributes,

$\{\delta_T\}$ - vector of random remainder (differences between transaction prices and model values).

The estimators of vector $[\hat{c}]$ and vector $\left[\hat{k}\right]$ may be written as following block matrices:

$$\begin{Bmatrix} [\hat{c}] \\ [\hat{k}] \end{Bmatrix} = \begin{Bmatrix} [S^T \cdot S] & [S^T \cdot a] \\ [a^T \cdot S] & [a^T \cdot a] \end{Bmatrix}^{-1} \times \begin{Bmatrix} [S]^T \\ [a]^T \end{Bmatrix} \times \{C_T\} \tag{4}$$

After including the estimators of vector $[\hat{c}]$ and vector $[\hat{k}]$, the vector of random remainder deviation $\{\hat{\delta}_T\}$ we can get from the following formula

$$\{\hat{\delta}_T\} = \{C_T\} - \{[S] \quad [a]\} \cdot \begin{Bmatrix} [\hat{c}] \\ [\hat{k}] \end{Bmatrix} \tag{5}$$

The remainder variance $(\hat{\sigma}_0^2)$ derived from parametric with taking weight coefficients into account will have the formula of

$$\hat{\sigma}_0^2 = \frac{\{\hat{\delta}\}^T \{\hat{\delta}\}}{j - R\{[S] \quad [a]\}}. \tag{6}$$

The covariance matrix of estimated parameters may be written in formula (7)

$$Cov \begin{Bmatrix} [\hat{c}] \\ [\hat{k}] \end{Bmatrix} = \hat{\sigma}_o^2 \begin{Bmatrix} [S^T \cdot S] & [S^T \cdot a] \\ [a^T \cdot S] & [a^T \cdot a] \end{Bmatrix}^{-1}. \tag{7}$$

Making the analysis of variance for transaction prices vector for whole estates (3) we may find the covariance matrix of $\{\hat{\delta}\}$ vector. It has the following form

$$Cov\{\hat{\delta}\} = \hat{\sigma}_0^2 \left(\{I\} - \{[S] \quad [a]\} \times \begin{Bmatrix} [S^T \cdot S] & [S^T \cdot a] \\ [a^T \cdot S] & [a^T \cdot a] \end{Bmatrix}^{-1} \{[S] \quad [a]\}^T \right). \tag{8}$$

3. THE VALUATION OF MARKET VALUE FOR URBAN ESTATES

If we valuate the industrial estate, which include parcel and buildings of known areas, these quantities may be written as the one row parameters' matrix

$$[\overline{S}] = [\overline{S}_1 \ \overline{S}_2 \ ... \overline{S}_i]. \tag{9}$$

The market attributes of valuated estate may be set together as the one row matrix, too

$$\left[\overline{a}\right] = \left[\overline{a}_1 \ \overline{a}_2 \ldots \overline{a}_j\right] \tag{10}$$

After taking into account estimated price coefficients (c_i) and weight coefficients (k_j), the estimated market value of elaborated urban estate can be expressed by the formula below

$$WR = \left\{\left[\overline{S}\right]\left[\overline{a}\right]\right\} \times \begin{Bmatrix} [\hat{c}] \\ [\hat{k}] \end{Bmatrix}. \tag{11}$$

The variance of estimated (using formula (11)) market value for urbanized estate can be defined including covariance matrix (7) as follows

$$\sigma^2(WR) = \left\{\left[\overline{S}\right]\left[\overline{a}\right]\right\} \times Cov\begin{Bmatrix} [\hat{c}] \\ [\hat{k}] \end{Bmatrix} \times \begin{Bmatrix} \left[\overline{S}\right]^T \\ \left[\overline{a}\right]^T \end{Bmatrix} \times \left\{\left[\overline{S}\right]\left[\overline{a}\right]\right\}^T. \tag{12}$$

The symmetric confidence interval for urban estate's estimated market value will be calculated from its standard deviation $(\sigma(WR))$ and the quantile of Student's distribution $(t(1-\alpha/2); u))$, for confidence level α and u degrees of freedom. It is presented in formula (13)

$$\vec{WR} = WR \pm \sigma(WR) \times t(1-\alpha/2; u) \tag{13}$$

4. THE EXAMPLES OF URBAN ESTATES' VALUATION

4.1 Market information

The application of valuation parametric model will be presented with the example of urban estates. Information from seven authenticated deeds obtained from public notaries was used here.

The following attributes has been taken into account during calculations:

- part of the city - suburbs,
- destination in land use city plan – the land of urbanization low intensification,
- localization – very good (2), good (1),
- access to parcel – good,
- vicinity – very good,
- public utilities – water, electricity, gas, sewage, road,
- standard of components used for building and decorating (building standard) – very high (2), high (1),
- usable area of urbanized buildings from 200 m^2 to 340 m^2,
- area of parcel from 810 m^2 to 1050 m^2.

There are seven urban real estates that were chosen for consideration. They description and attributes are listed below.

1. Built up estate with commercial building. Building usable area is 260 m^2. Parcel area is 850 m^2. Attributes: localization – very good (2), building standard – high (1), price – 1 570 000 zł (402 000 EUR 1 EUR = 3.9 zł)
2. Built up estate with commercial building. Building usable area is 300 m^2. Parcel area is 970 m^2. Attributes: localization – good (1), building standard – high (1), price – 1 600 000 zł (410 000 EUR)
3. Built up estate with commercial building. Building usable area is 220 m^2. Parcel area is 760 m^2. Attributes: localization – very good (2), building standard – very high (2), price – 1 450 000 zł (372 000 EUR)
4. Built up estate with commercial building. Building usable area is 320 m^2. Parcel area is 910 m^2. Attributes: localization – very good (2), building standard – high (1), price – 1 800 000 zł (462 000 EUR)
5. Built up estate with commercial building. Building usable area is 200 m^2. Parcel area is 810 m^2. Attributes: localization –good (1), building standard – high (1), price – 1 200 000 zł (308 000 EUR)
6. Built up estate with commercial building. Building usable area is 340 m^2. Parcel area is 1050 m^2. Attributes: localization – very good (2), building standard – high (1), price – 1 900 000 zł (487 000 EUR)
7. Built up estate with commercial building. Building usable area is 290 m^2. Parcel area is 880 m^2. Attributes: localization – good (1), building standard – high (1), price – 1 570 000 zł (397 000 EUR).

There are two factors influencing estates' prices in the examples given above. They are localization and building standard.

4.2 The estimation of valuation model's parameters

The information obtained from deeds is basis for 7 equations written according to formula (1). Every equation will have two price coefficients and two weight coefficients. The matrix [S] of parcel areas and building usable areas and the matrix [a] of market attributes and the matrix $\{C_T\}$ of transaction prices have the following form:

$$[S] = \begin{bmatrix} 850 & 260 \\ 970 & 300 \\ 760 & 220 \\ 910 & 320 \\ 810 & 200 \\ 1050 & 340 \\ 880 & 290 \end{bmatrix} \quad [a] = \begin{bmatrix} 2 & 1 \\ 1 & 1 \\ 2 & 2 \\ 2 & 1 \\ 1 & 1 \\ 2 & 1 \\ 1 & 1 \end{bmatrix} \quad \{C_T\} = \begin{Bmatrix} 1570000 \\ 1600000 \\ 1450000 \\ 1800000 \\ 1200000 \\ 1900000 \\ 1550000 \end{Bmatrix}$$

From the estimation of this model's parameters according to formulas (4) ÷ (8) unit prices coefficients and random remainders, calculated for every transaction were obtained. The unit parcel and urban buildings' prices coefficients are as follows:

$\hat{c}_G = 349,43\ zł/m^2$, $\hat{c}_B = 3547,27\ zł/m^2$, $k_1 = 137789$, $k_2 = 65959$.

The random remainders (differences between transaction prices and model values) are presented in the table below

Transaction number	Random remainder δ_{Tj}	Calculations
1	9 157	$\sum_{i=1}^{7} \sigma_i^2 = 4973950134$
2	-6 817	
3	-3 463	
4	5 355	
5	3 759	
6	-14 511	
7	10 044	

The remainder variance ($\hat{\sigma}_0^2$) obtained from Gauss – Markov model estimation and the covariance matrix ($Cov\{\hat{c}\}$) for unit prices coefficients have values as follows

$\sigma_0^{\ 2} = 1657983378$

$\sigma_0 = 12\ 876$

The covariance matrix obtained for all estimated parameters have the following values

$$Cov\left\{\begin{bmatrix}\hat{c}\\ \hat{k}\end{bmatrix}\right\} = 1657,983\begin{bmatrix} 31.02 & -92.34 & 1561.85 & -3856.94 \\ /-/ & 295.00 & -7277.59 & 10584.97 \\ /-/ & /-/ & 898922.81 & -695651.17 \\ /-/ & /-/ & /-/ & 1424580.04 \end{bmatrix}$$

The model's estimated parameters presented above are basis for the market value of similar estates estimation.

4.3 The valuation of similar estates

As a subject of valuation we choose estate similar to estates elaborated in market analysis presented above. This is the commercial estate, which has usable area of 260 m^2. The parcel it

is situated at has area of 980 m^2. The localization is good (1) and building standard high (1). Applying formula (11) we obtained estates market value as follows:

$$WR = \left\{\left[\bar{S}\right]\left[\bar{a}\right]\right\} \times \begin{Bmatrix}\left[\hat{c}\right] \\ \left[\hat{k}\right]\end{Bmatrix} = \begin{bmatrix}980 & 260 & 1 & 1\end{bmatrix} \times \begin{bmatrix}349,43 \\ 3547.27 \\ 137789 \\ 65959\end{bmatrix} = 1\ 468\ 480\ zł$$

.

After performing operation on the proper matrices, the standard deviation of estimated market value was calculated below:

$$\sigma^2(WR) = 1657,983 \times \begin{bmatrix}980 & 260 & 1 & 1\end{bmatrix} \times \begin{bmatrix}31.02 & -92.34 & 1561.85 & -3856.94 \\ /-/ & 295.00 & -7277.59 & 10584.97 \\ /-/ & /-/ & 898922.81 & -695651.17 \\ /-/ & /-/ & /-/ & 1424580.04\end{bmatrix} \times \begin{bmatrix}980 \\ 260 \\ 1 \\ 1\end{bmatrix}$$

$$\sigma^2(WR) = 1377460000 \ \Leftrightarrow \ \sigma(WR) = 37\ 114\ zł$$

Applying Student distribution quantile for 3 degrees of freedom and confidence level $(1-\alpha) = 0.95$, that means $t(0.95;\ 3) = 3,2$, the symmetric confidence intervals for estimated estate's market value equals

$$\tilde{WR} = 1\ 468\ 480 \pm 118\ 765\, zł\ .$$

The variance analysis, which was made in the example presented above shows that estimated estate's market value has confidence interval with range about 16% its value.

5. CONCLUSIONS

All information obtained from estates' market should be subject of probabilistic modeling. This modeling ought to lead to the estimation of the most probable unit coefficients of elaborated estates' elements' prices and attributes weight coefficients.
In these parameters estimation process, the complete variance analysis including confidence intervals evaluation ought to be performed.
The urban estates' market value estimation should always be performed with complete preciseness evaluation.

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