NATIONAL TECHNICAL UNIVERSITY OF ATHENS
SCHOOL OF RURAL AND SURVEYING ENGINEERING
DEPARTMENT OF TOPOGRAPHY
LABORATORY OF GENERAL GEODESY

Assessing the use of "light" laser scanners and the Monte Carlo technique for the documentation of geometric surfaces

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FIG Working Week 2011
Bridging the Gap between Cultures
Marrakech, Morocco, 18-22 May 2011

The main targets

- A-priori estimation of the measured points' uncertainty by using the "light" laser scanners.
- Selection of the proper total station and the appropriate scanning distance according to the desired uncertainty result.
- Determination of the scanning parameters such as the scanning steps (horizontally and vertically) and consequently the maximum number of points to be measured, as well as the a-priori standard error of a geometric surface adjustment.

Monte Carlo Technique + Least square Method

"light" laser scanners

- total stations (servo or piezoelectric driven)
- No support of external batteries and individual lap-top is needed
- Convenient for geometrical surface documentation
- horizontal and vertical scanning step
- Individual point measurement

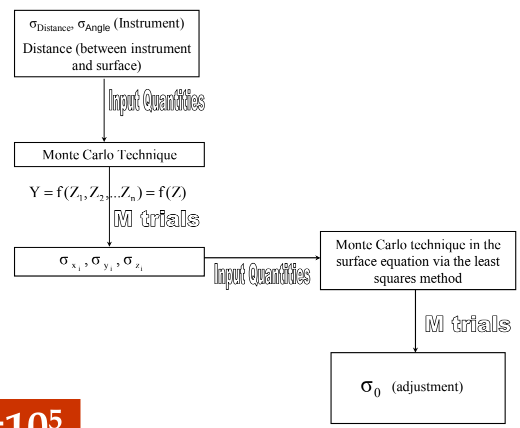
Total Station	Distance accuracy (RL)	Angle accuracy	Max. Scan speed	Range (m)
Topcon IS	±5 mm	±0.3mgon	20 points/sec	<250
Trimble VX	±3 mm ±2 ppm	±0.3mgon	15 points/sec	<150

THE A-PRIORI SURFACE ADJUSTMENT

$$\sigma_{x_i} = \sigma_{y_i} \approx \pm\sqrt{\sigma_{D_i}^2 + \frac{D_i^{\,2}}{(\rho^{cc})^2}\cdot\left(\sigma_z^2 + \sigma_\alpha^2\right)}$$

$$\sigma_{z_i} \approx \pm\sqrt{\sigma_{D_i}^2 + D_i^{\,2}\cdot\left(\frac{\sigma_z}{\rho^{cc}}\right)^2}$$

Monte Carlo technique



$M=10^5$

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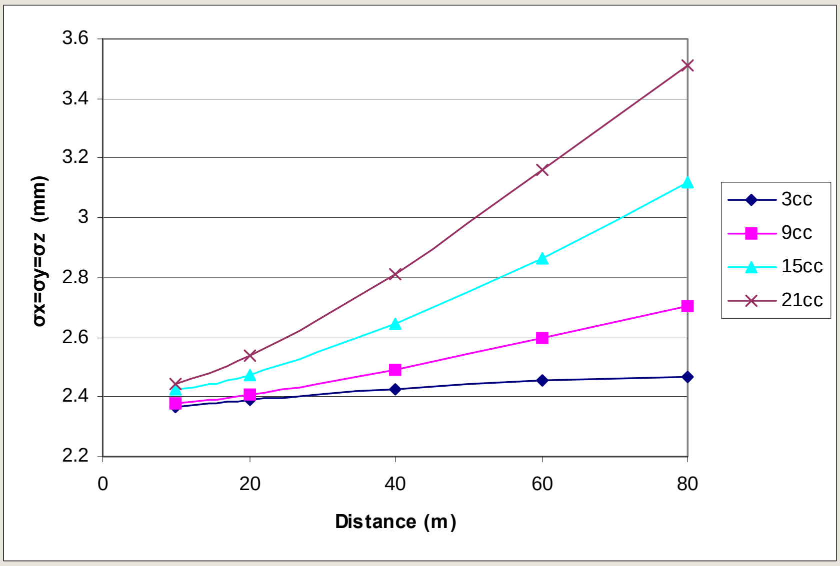
The σ_x, σ_y, σ_z values for different instruments, for distance measurement with an accuracy of ±3mm



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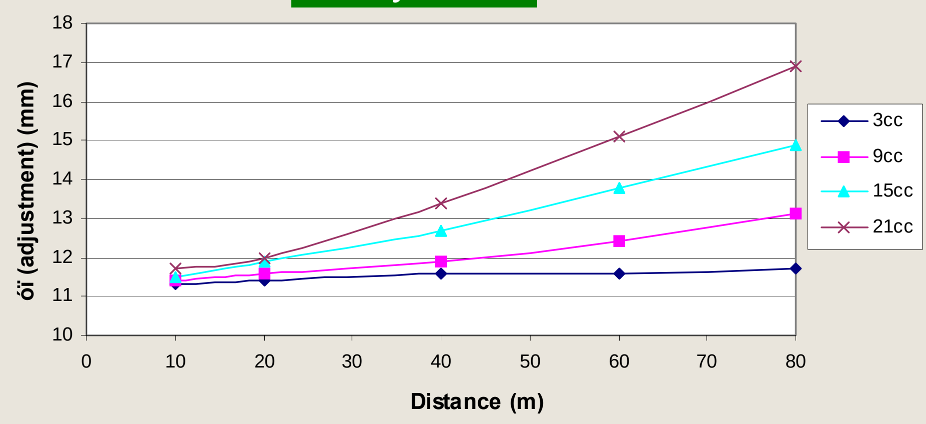
The a-priori standard error σ_0 for the adjustment of a plan

$$5x+8y-2z=0$$



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how many points have to be scanned on a concrete surface ?

σ_0

same step, horizontally and vertically

$$\text{step } s = \sigma_0 \cdot z_{95\%}$$

the approximate number **n** of the points

$$n = \frac{\text{Area}}{s^2}$$

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THE A-POSTERIORI SURFACE ADJUSTMENT

- least square method is used
- The unknown surface is expressed by a linear or no linear function of the calculated x, y *z*
- The unknown parameters are the coefficients **ai**

$$A \cdot \hat{x} = l + u$$

$$\hat{x} = \left(A^T \cdot A\right)^{-1} \cdot A^T \cdot l$$

$\sigma_{a\text{-posteriori}}$

$$V_x = \begin{bmatrix} \sigma_{a_1}^2 & & & \\ & \sigma_{a_2}^2 & & \\ & & \ddots & \\ & & & \sigma_{a_m}^2 \end{bmatrix}$$

Where

$$A = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \dots & \alpha_{1m} \\ \alpha_{21} & \alpha_{22} & \dots & \alpha_{2m} \\ \vdots & \dots & \ddots & \vdots \\ \alpha_{n1} & \alpha_{n2} & \dots & \alpha_{nm} \end{bmatrix}$$

$$\hat{x} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{bmatrix}, l = \begin{bmatrix} l_1 \\ l_2 \\ \vdots \\ l_m \end{bmatrix}$$

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Statistical Checks

1st $\acute{o}_{a_i} \cdot z_{95\%} \le a_i$

If equation is valid for all the coefficients then the surface is **acceptable**

2nd $\acute{o}_{a\text{-posteriori}} \le \acute{o}_{a\text{-priori}} \cdot z_{95\%}$

If equation is valid then the **measured points belong to the geometric-mathematic model** and hence **the surface has no manufacture errors**

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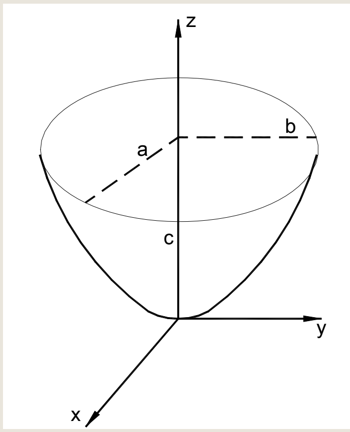
APPLICATION

satellite antenna



$$\frac{(x_i - x_0)^2}{a^2} + \frac{(y_i - y_0)^2}{b^2} = \frac{1}{c} \cdot (z_i - z_0)$$

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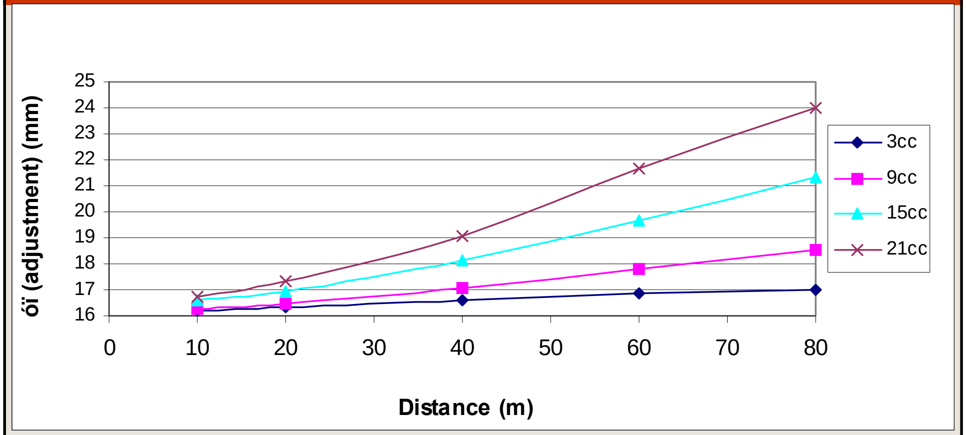
The determination of the a-priori parameters



- Using a total station $\pm 3^{cc}$ angle accuracy
- Distance = 10m

$\sigma_0 = 1.6\text{cm}$

$s = 1.6 \cdot z_{95\%} = 3.2\text{cm}$

~ 1000 points/m²

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The determination of the a-posteriori surface

Trimble VX

scanning step = 4cm

horizontally and vertically

2600 points

a(m)	σ_a (mm)	b(m)	σ_b (mm)	x_0 (m)	σx_0 (mm)	y_0 (m)	σy_0 (mm)	z_0 (m)	σz_0 (mm)
0.951	±0.3	0.965	±0.3	0.001	±0.0001	-0.019	±0.0002	0.001	±0.0001

- $\pm 3^{cc}$ for the directions
- $\pm 3\text{mm} \pm 3\text{ppm}$ for distance

$\sigma_0 = \pm 12\text{mm}$

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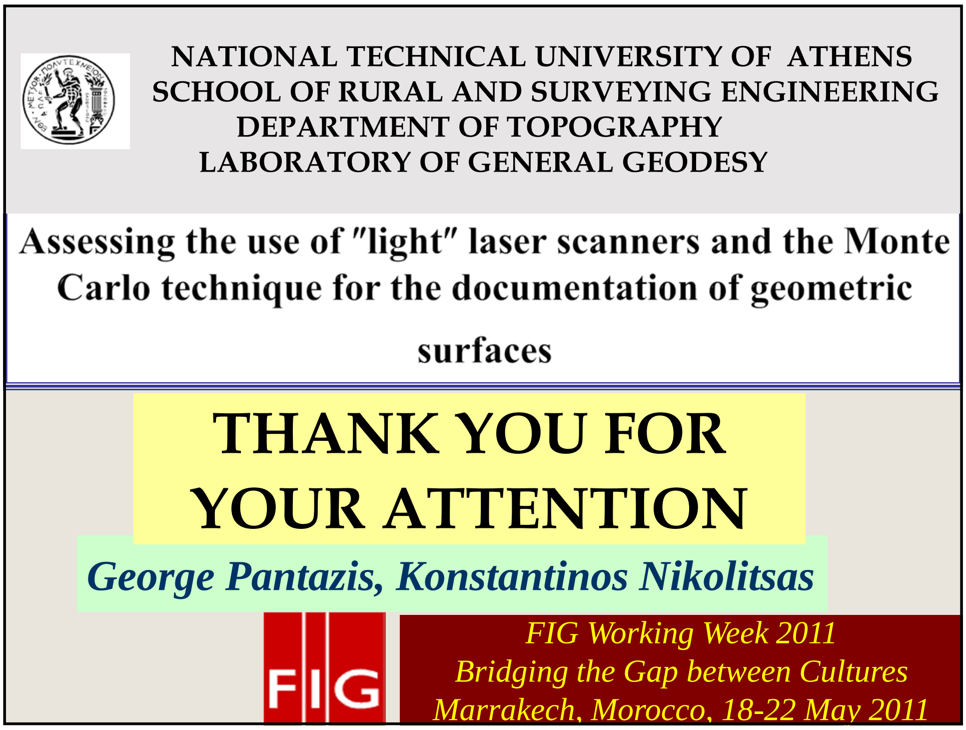
REMARKS AND CONCLUSIONS

- The **a priori standard error** of the points' adjustment, which belong to a specific surface is strongly influenced by the accuracy that the total station provides and can estimated by using the **Monte Carlo technique.**
- Knowing the number of points, which are necessary to be captured and the desired σ_0 of the adjustment, the user can have a better understanding of what he needs to collect at the field
- The **comparison** between the a-priori and the a-posteriori σ_0 of the adjustment can document that the measured surface is **constructed according** to its specifications.

REMARKS AND CONCLUSIONS

- The **Monte Carlo** technique proved to be a very useful tool for the **a-priori determination** of the measurements' uncertainty as well as the standard error of the adjustment.
- The development of **total stations**, with laser scanner capability, gives the opportunity for a more **economical** procedure of scanning geometric surfaces compared with the real laser scanner.
- These instruments are more **convenient** when processing data when compared to the laser scanners, as a less bulky computer is needed. They are more **easy to use** as they are lighter and have the same on board software as the conventional surveys.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THANK YOU FOR YOUR ATTENTION
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