An Empirical Investigation of the Effect of Sea Level Variation on Vertical Reference Frames: Case study of Lagos state, Nigeria.

ODUMOSU, Joseph O.
ABIODUN, O. A
KEVIN, K. M.
ADAMU, G. M

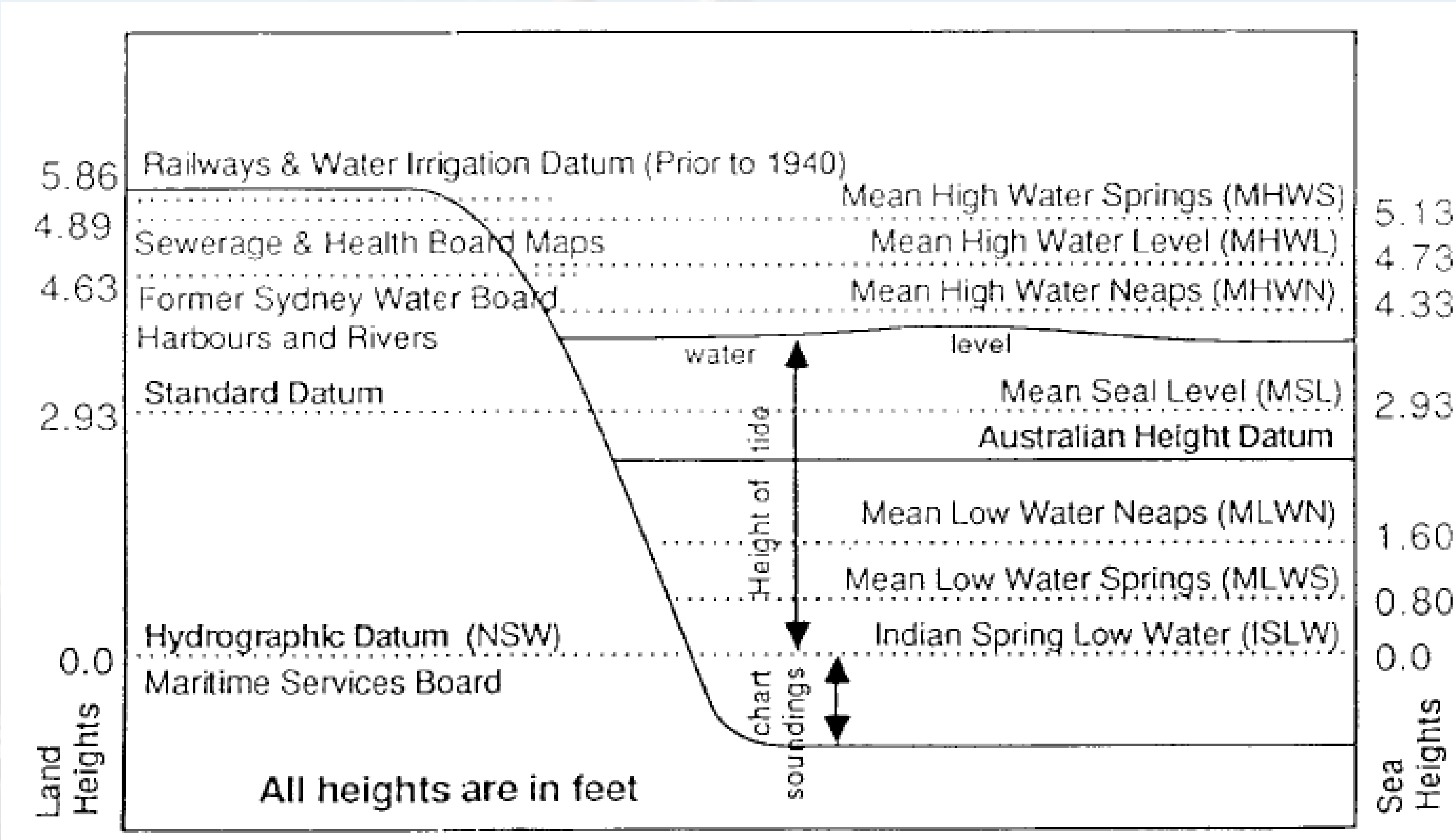
INTRODUCTION

- Most Environmental Hazards are spatially related.

- Extent of spread of such hazards could be influenced by terrain undulation or configuration.

- Terrain undulation refers to the height variation of an area of land, depicting the low and high lands in the area with reference to a reference measurement surface

INTRODUCTION (Contd)



SOME VERTICAL DATUMS USED FOR HEIGHT DETERMINATION (Rizos, 2015)

INTRODUCTION (Contd)

- In the real sense (as geodesists) “height is the distance of a point above a specified surface of constant potential; the distance measured along the direction of gravity”
- $$H = {(W_0 - W_P)}/{g} \qquad (1)$$

Where:

g = gravity Vector

W_0 and W_P = Potential at Reference Point and Observation Point resp.

- Such surface of constant potential is called an equipotential surface and the geoid is one of such many equipotential surfaces of the earth’s gravity field.
- However, the “near-coincidence” of the MSL with the geoid has over the years popularized the adoption of sea-related heights (Tide Gauge Datum) for the realization of several national vertical reference frames

INTRODUCTION (Contd)

Climate change has continued to cause the Mean Sea Level to rise by several centimeters over the years.

This research therefore looks at the effects of MSL variation on Lagos vertical Reference Frame work from the geodetic perspective by taking advantage of the direct relationship between heights and earth surface potentials.

EFFECT OF SEA LEVEL RISE ON VERTICAL REFERENCE FRAME

- Changes in sea Level with time results in a shift in the origin of the Vertical Reference Frame (VRF) because the equipotential surface used as the zero height reference (MSL) will change in position.

- This might have a constant effect at the origin , its effect on individual points within the frame work will vary due to the non-parallelism of the equipotential surfaces.

$$H_t - H_i = \frac{\delta W_0}{g_t} - \frac{\delta g}{\delta H} \cdot \frac{\delta W_0{}^2}{2 g_t{}^3} + \ldots\ldots \quad (2)$$

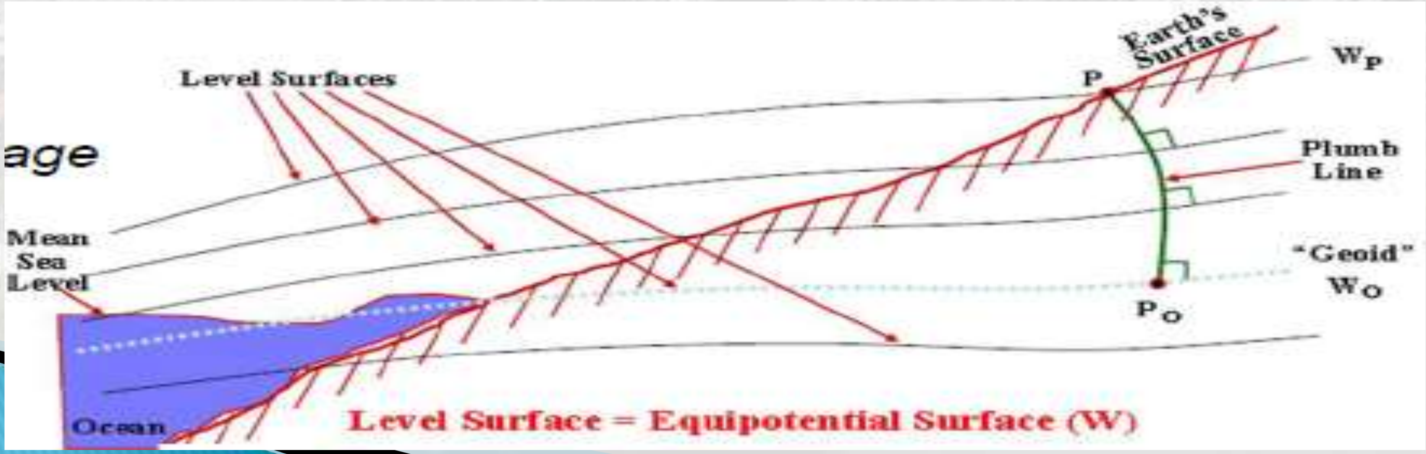


Figure 2: Height from a Gravimetric Perspective

METHODOLOGY / MODEL FORMULATION

- Due to lack of absolute gravity data for the study area, actual gravity is replaced in equations 1 and 2 with normal gravity since normal gravity potential accounts for approximately 99.9995% of the total potential (Jekeli, 2007).

- Since Potentials and normal gravity values are position dependent and do not change with time, equation 1 is used to determine station potential given the station orthometric height after which the orthometric height is recomputed using the station potential and a new reference potential.

- The classical Gauss Marcov functional model was used in this study to determine the height implication of a 50cm sea Level variation on the Vertical Rererence Framework within the study area.

$$C_v = \sigma^2{}_0 Q \qquad (3)$$

MODEL FORMULATION

Where:

$$\sigma^2{}_0 = {V^T PV}/{r} \qquad (4)$$

Where:

V= Observational Residuals

r= Number of conditions

$$Q_{L^a} = P^{-1} - P^{-1}B^T M^{-1} B P^{-1} \qquad (5)$$

$$[(H_2 - H_1) + (H_3 - H_2) + (H_1 - H_3)]_p = [(H_2 - H_1) + (H_3 - H_2) + (H_1 - H_3)]_n = 0 \quad (6)$$

But

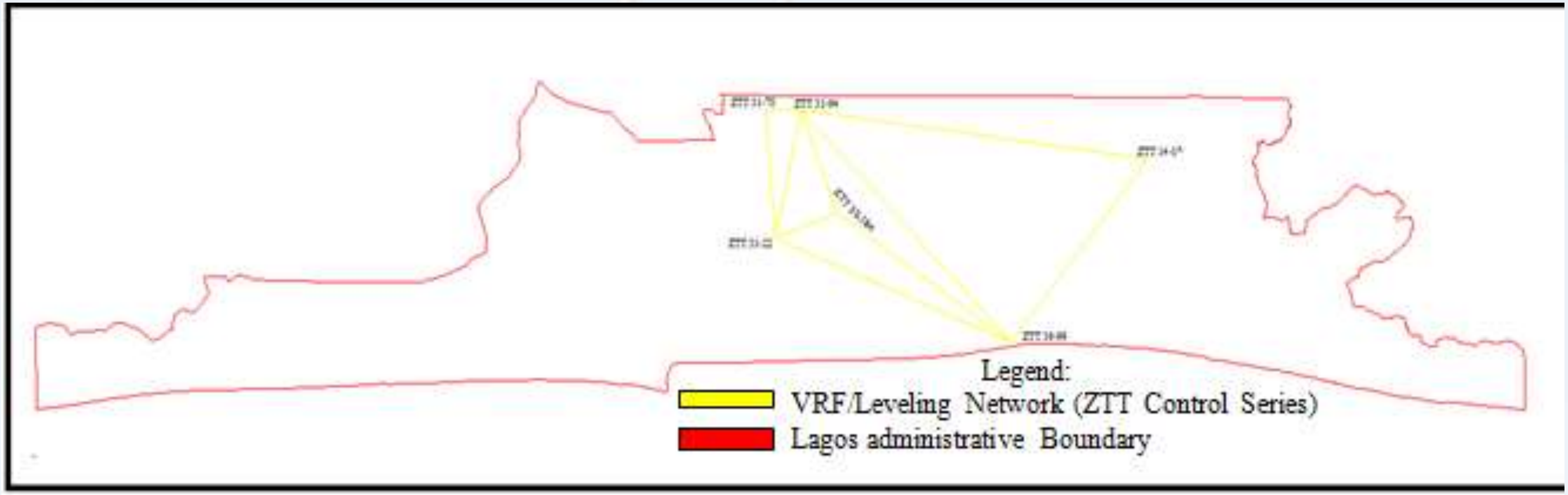
$$[H_2 - H_1]_p = \Delta h_{1_p} \qquad (7a)$$

$$[H_2 - H_1]_n = \Delta h_{1_n} \qquad (7b)$$

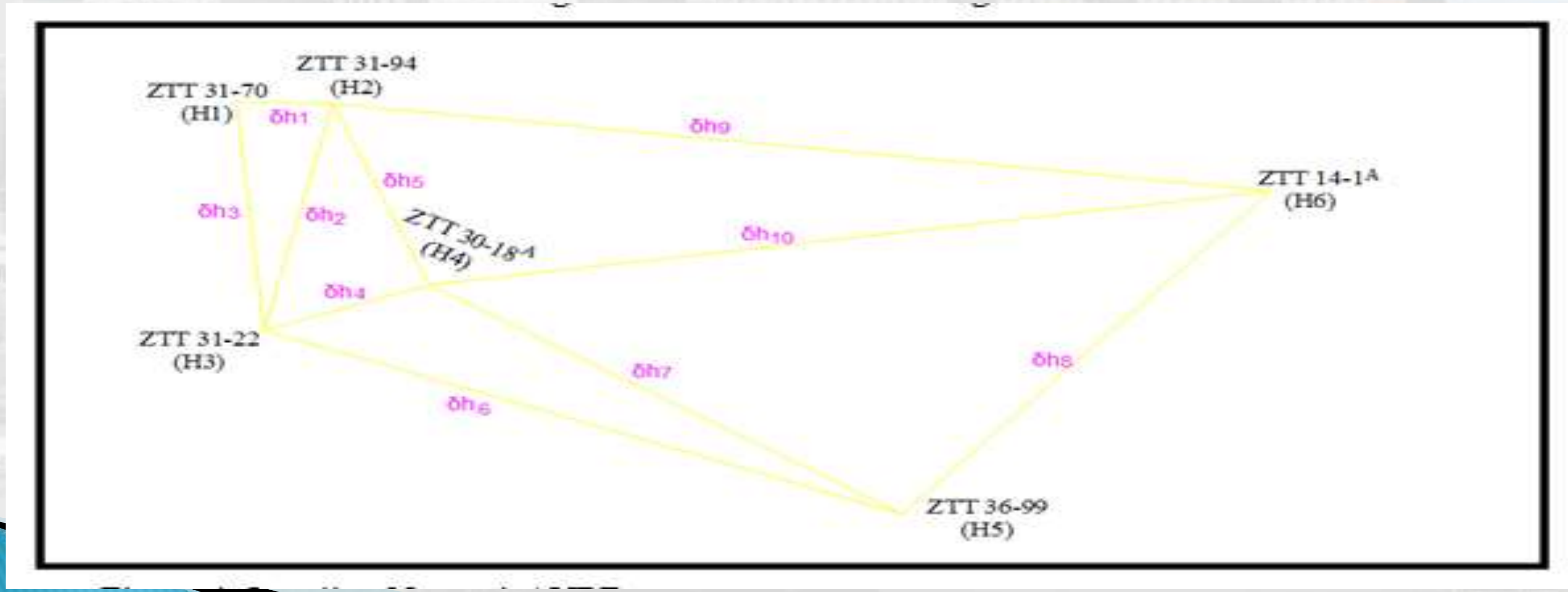
$$\therefore \Delta h_{1_p} - \Delta h_{1_n} = \delta h_1 \qquad (8)$$

Equation (6) then becomes (9)

$$\delta h_1 + \delta h_2 + \delta h_3 = 0 \qquad (9)$$



STUDY AREA (figure 3)



Lagos Leveling Framework used for study. (Figure 4)

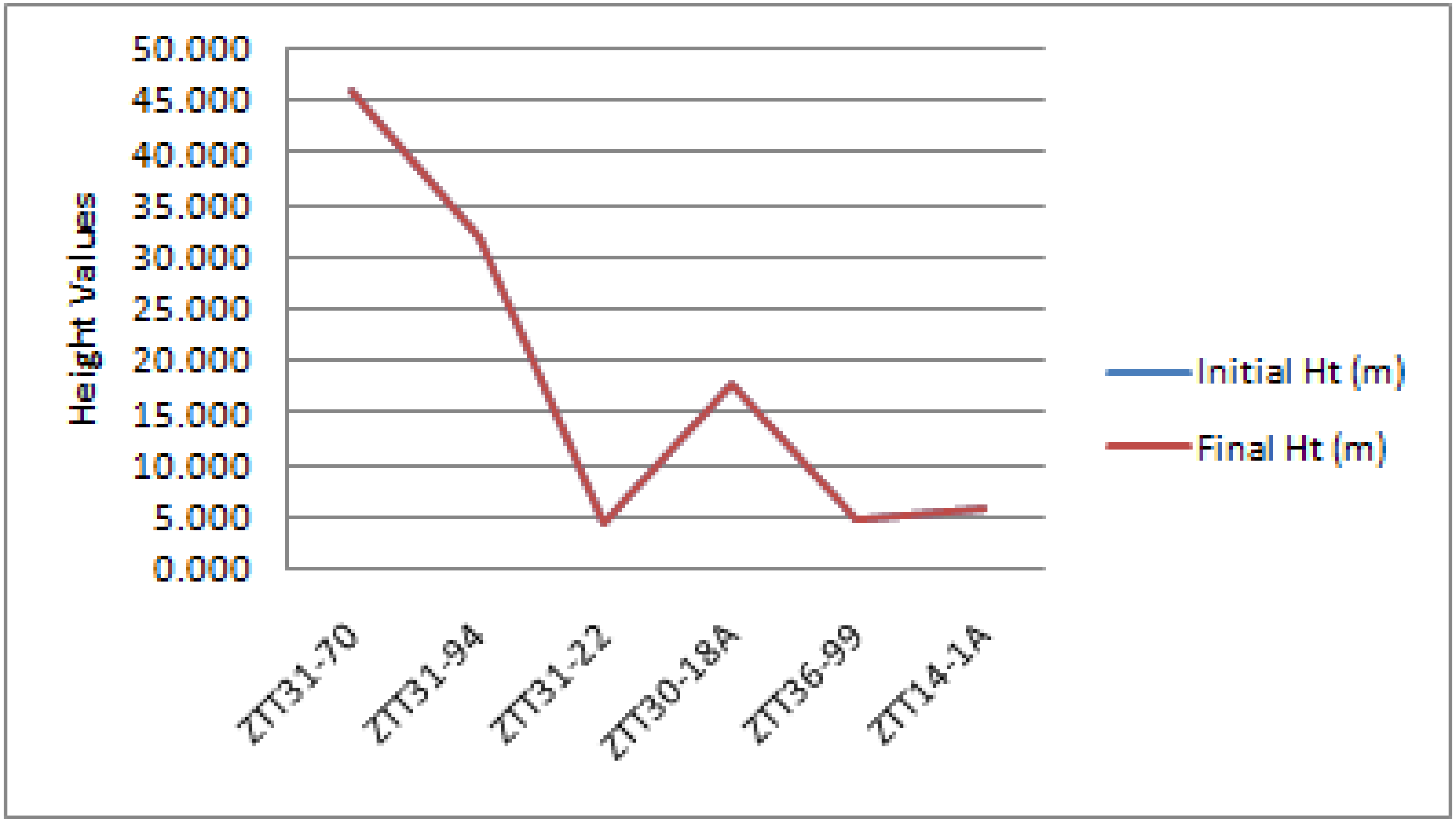
DATA USED (Table 1)

S/N	DATA	SOURCE	ACCURACY
1.	Initial Orthometric Heights (H_p)	Office of the Surveyor General of Lagos State – Interspatial Surveys. (6 points within the 2nd Order State-wide Controls Network)	2nd – Order Accuracy
2.	Normal Gravity	Computed using Equation (10)	N/A
3.	Station Geo-potential	Computed using Equation (9)	N/A
4.	Final Orthometric Heights (H_n)	Computed using Equation (9)	N/A

RESULTS (Table 2)

STATn. ID	Initial Wo (m2/s2)	Final Wo (m2/s2)	Station Potential	Initial Ht (m)	Final Ht (m)	Residual
ZTT31-70	62 636 860.0	62 636 854.0	17640164.58	46.002	46.001970	-0.000030
ZTT31-94	66 636 860.0	66 636 854.0	31558211.47	31.773	31.772977	-0.000023
ZTT31-22	63 636 860.0	63 636 854.0	58366291.08	4.366	4.365995	-0.000005
ZTT30-18A	64 636 860.0	64 636 854.0	45288512.01	17.736	17.735998	-0.000002
ZTT36-99	65 636 860.0	65 636 854.0	58010263.45	4.730	4.729993	-0.000007
ZTT14-1A	67 636 860.0	67 636 854.0	56825701.69	5.941	5.940993	-0.000007

RESULTS (Figure 5): Plot of heights of observed heights before and after MSL rise.



RESULTS (Figure 6): Plot of spatial pattern of the magnitude of height residuals observed at the investigated stations before and after MSL rise.

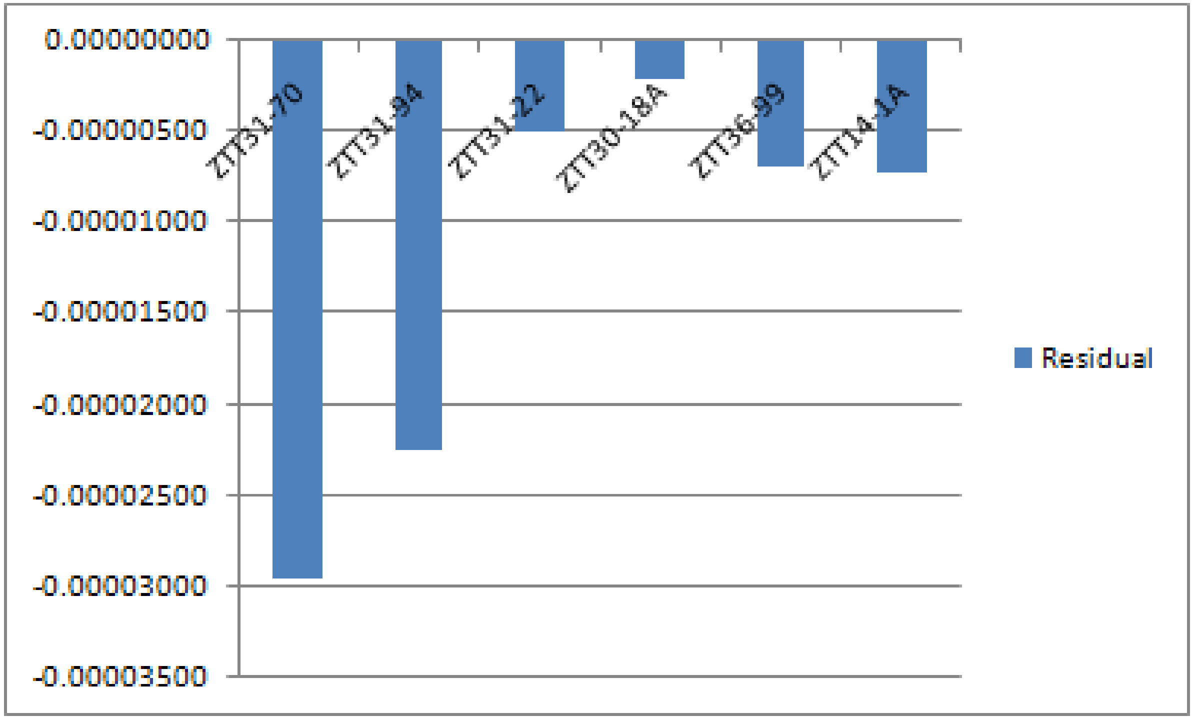


Figure 7: Plot of spatial pattern of the magnitude of residual between Heights of selected control points before and after MSL rise.

RESULTS (Table 3a): Summary of Results

Measured Parameter (Among Baselines)	Value (m)
Standard Deviation	∓ 0.000000000015
RMSE	0.00000000000299
Variances along bases $\begin{bmatrix} \delta h_1 \\ \delta h_2 \\ \delta h_3 \\ \delta h_4 \\ \delta h_5 \\ \delta h_6 \\ \delta h_7 \\ \delta h_8 \\ \delta h_9 \\ \delta h_{10} \end{bmatrix}$	$\begin{bmatrix} -0.00000000283 \\ -0.00000000388 \\ -0.00000000567 \\ -0.00000000373 \\ -0.00000000865 \\ -0.00000000626 \\ -0.00000000537 \\ -0.00000000328 \\ -0.00000000313 \\ -0.00000000492 \end{bmatrix}$

RESULTS (Table 3b): Summary of Results (Contd)

Measured Parameter (Among Baselines)	Values (m)
Minimum Deviation along base lines (δh_1)	−0.00000000283
Maximum Deviation along base lines (δh_5)	−0.00000000865

DISCUSSION OF RESULTS

- All residuals in the heights are negative. This trend is expected as a rise in MSL will lead to inundation of land masses.

- As seen however in Figure 6, the pattern of the inundation is irregular; confirming the theory of non-parallelism of the equipotential surfaces.

- Control points ZTT36-99 and ZTT 14-1A which both lie close to the Lagoon (about same equipotential surface) experienced similar amount of inundation signifing the dependence of heights on geo-potential differences.

DISCUSSION OF RESULTS (Contd)

- ZTT31-70 and ZTT31-94 which are located far from the shoreline/coast area however experienced a greater inundation than some coast-line controls; signifying a possibility of greater effects of MSL variation on interland points than in coastal areas.

- It can however be seen from the gauss marcov functional statistical model that the maximum effect of the MSL variation in the VRF is noticeable along the baseline dh5 and minimum variation along baseline dh1.

CONCLUSION

- It has been verified from the gauss marcov functional model analysis that the effect of MSL variation on VRF is non-linear i.e does not dependend on distances between baselines.

- It therefore can be concluded that the variation of the MSL has a minimal effect (0.000000000015m) on the VRF within the study area. As such, the changing MSL does not pose any danger to the suitability of the existing vertical controls within the study area.

- THANKS FOR LISTENING