An Entire Spectrum Modernization of the Geoid Model

---- From data collection to modeling and customer services

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SUMMARY

Determining the geometric shape, orientation in space, and gravity field of the Earth is the main objective of geodesy, the science that supports geomatics, land surveying, and many civil engineering applications. An accurate geoid model with high resolution is fundamental to these goals. Evolving from its leveling-based vertical datum, the U.S. National Geodetic Survey (NGS) is updating its vertical datum to a geoid-based one, which is nominally called the NAPGD2022 (North American-Pacific Geopotential Datum of 2022). This paper summarizes the primary efforts toward modernizing the Geopotential Datum, which includes new gravity data collections, modeling methodology updates, model validations, and product distributions. While relying on satellite gravimetric missions for the long wavelengths, NGS launched the GRAV-D (Gravity for the Redefinition of the Vertical Datum) project to collect airborne gravity data that homogeneously cover the entirety of the U.S., including territories and protectorates, with an extension of about 200km to its neighboring countries and open ocean areas along the coastlines. This new data calls for a full-spectrum modernization of the geoid modeling and computation procedures to optimally combine it with signals from satellite geopotential models, surface gravity surveys, high-resolution digital elevation models, and satellite altimetry data. A series of research tasks have been ongoing, including critical steps such as downward continuation, topographic modeling, and developing local functions. Several validation tests on land and multi-year/multi-mission averaged water surface heights are used to evaluate the geoid models.

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1. INTRODUCTION

Throughout history, people have needed to know their heights above sea level; see Smith (1992, 1996) for accurate definitions of various heights. Geodetic leveling using spirit leveling instruments (Young, 1989) and gravity measurements were widely used to obtain Helmert orthometric heights (Heiskanen and Moritz, 1967). Because of its intuitiveness, the resulting leveling networks were often adopted as the vertical datum, such as the widely used North American Vertical Datum of 1988 (NAVD 1988; Zilkoski et al., 1992). These kinds of vertical datums are not derived from the geodetic boundary value problem (GBVP) or have analytical forms, and they should not be called models, strictly speaking. Moreover, the accumulated errors along the leveling routes often generate salient systematic errors shown in Fig. 1 (Wang et al., 2012). Latent variable analysis techniques must be used to reveal the underlining reasons that caused these errors (Li, 2018a).

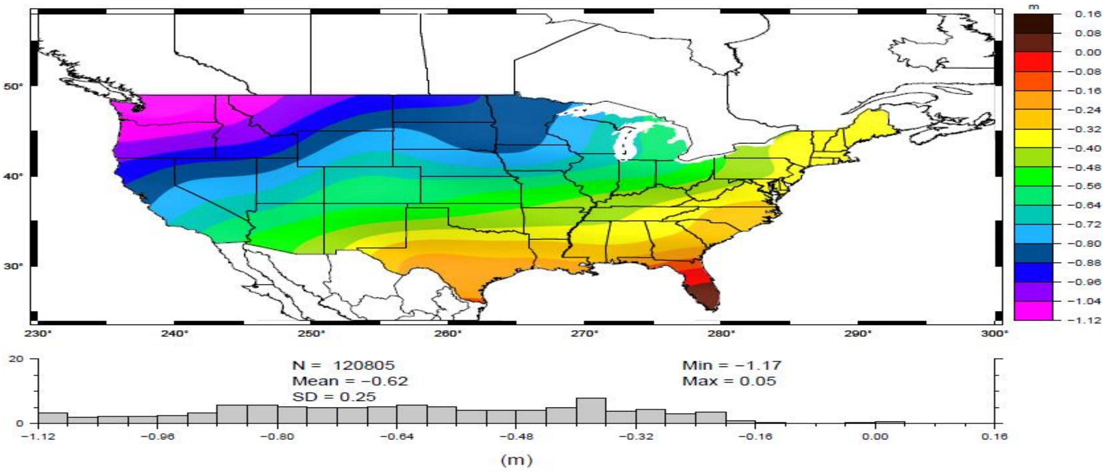

Figure 1. Meter-level systematic vertical datum errors over CONUS area (lower panel is the histogram of the errors) (Wang et al., 2012)

A geoid model is more suitable to serve as the vertical datum. The first gravimetric geoid model by the National Geodetic Survey (NGS) was the GEOID90 model created by Milbert (1991). Following this model, continuous efforts were made to update the geoid model approximately every three years (Roman et al., 2010a/b). Significant theoretical changes were made while developing the USGG2009 (Wang et al., 2012) to take advantage of high-degree and order global reference models, such as the EGM2008 model (Pavlis et al., 2012).

Because of the rapid increase in airborne gravity coverage from the GRAV-D project (Smith, 2007), in 2014, NGS released annual rather than triennially updated geoid models (Roman and

Li, 2014). In addition to the new airborne gravity data, many parts of the algorithms are spruced up to make the techniques more rigorously aligned with the analytical solutions of the GBVP. This paper summarizes these updates both in data availabilities and algorithm augmentations. Section 2 highlights the GRAV-D contributions and the DEM updates. Section 3 describes some major tests on algorithm updates, such as research done on harmonic downward continuations, the use of more precise mass elements and density models in topographic modeling, the use of local functions to capture local gravity field, and the impacts of harmonic corrections in the scenarios of filling up masses in the valleys that are below the reference surface, as well as a more accurate geoid to quasi-geoid separation term (Wang et al., 2023). Finally, a summary is given in Section 4. Considering that most of the research is already published or will be published in individual manuscripts, this paper intends to provide a high-level overview to facilitate a quick understanding of the big picture related to the geoid model modernization without going into the mathematical details. However, comprehensive references are given for interested readers to discover the intricacies of their implementations.

2. GRAVITY and ELEVATION DATA UPDATE

The GEOID90 model used about 1.5 million surface and shipborne gravity data points (Milbert, 1991). Saleh et al. (2013) separated these gravity data into 1,489 surveys and found that 244 have significant biases (> 2 mGal), introducing geoid errors up to 20 cm. These are shown in Figure 2 and Figure 3, respectively. In addition, they found that high-frequency errors of about 2.2 mGal contaminate the data, by using crossover analysis and K-nearest neighbor predictions. The resulting geoid errors are shown in Figure 4. An updated parametric estimation method is developed to determine the local biases in rigorous least squares (Li, 2021). Figure 5 shows that this new method can fix the un-detected errors in the previous results, as shown in Figure 2.

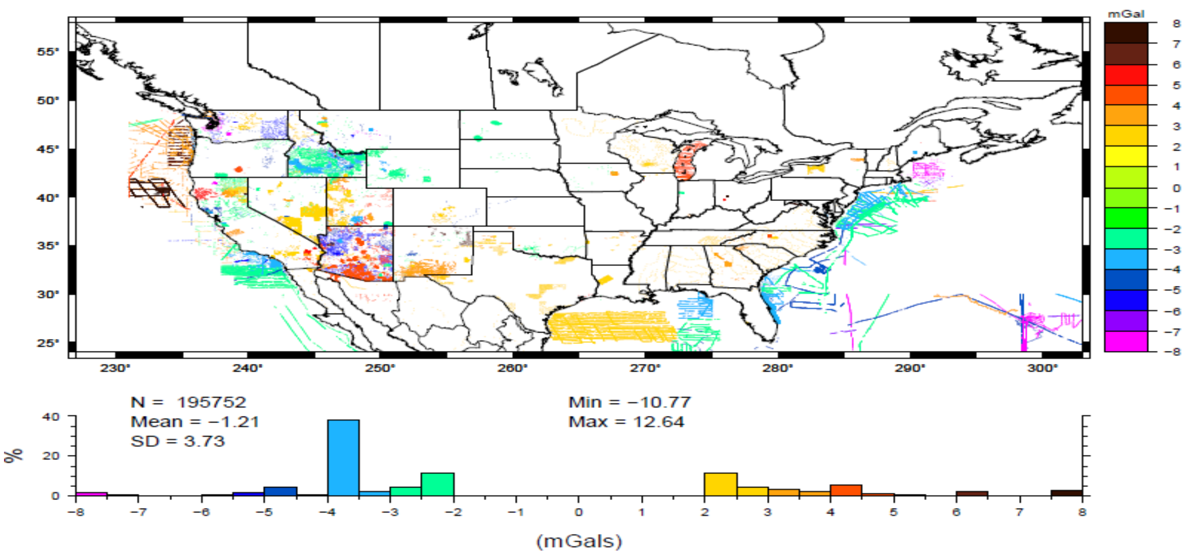

Figure 2. A total of 244 biased surveys in the NGS surface gravity data; Saleh et al. (2013)

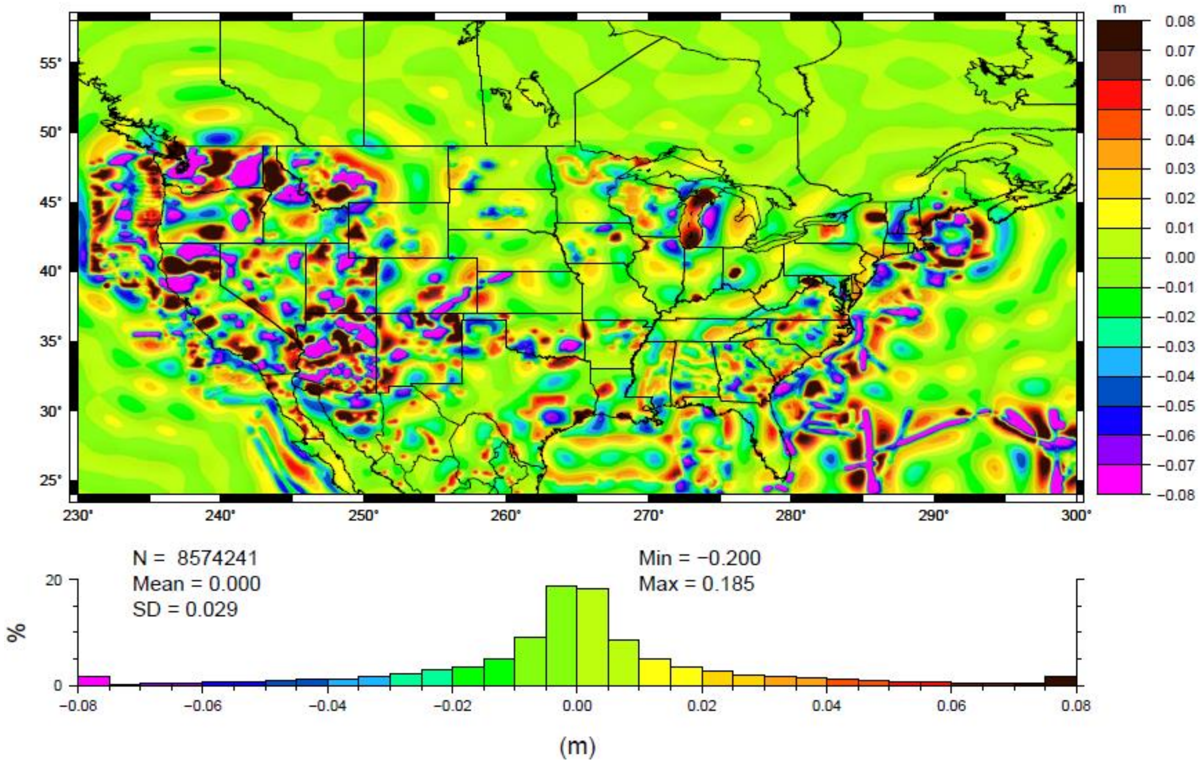

Figure 3. The resulting geoid errors are caused by the data shown in Figure 2; Saleh et al. (2013).

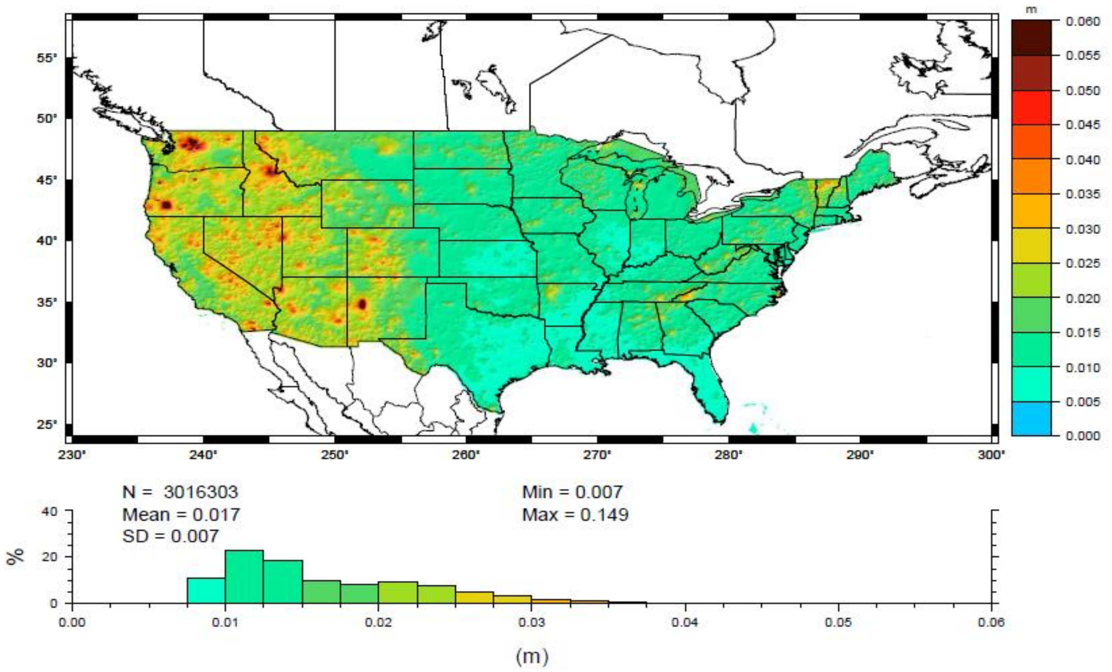

Figure 4. Geoid errors due to the high-frequency noise in the gravity data; Saleh et al. (2013).

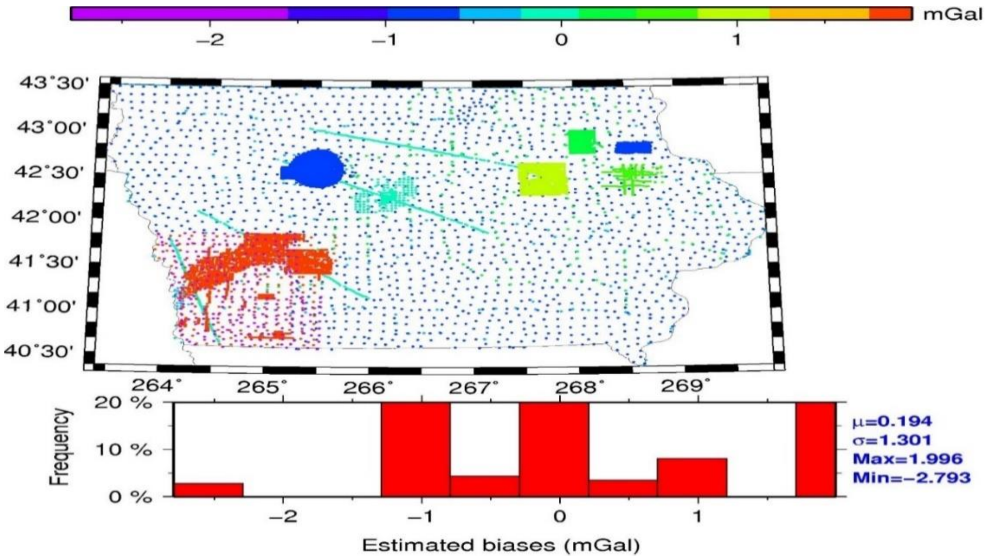

Figure 5. The rigorous parametric estimation method detected survey biases in the gravity data neglected in the previous results; Li (2021).

NGS employs airborne gravimetry (Childers et al., 1999) to collect gravity data efficiently and consistently. While airborne gravity data necessitates a downward continuation with all its problems (Li et al., 2022), it is more or less uniformly distributed and tends to have consistent stochastic properties. For example, data collected by adjacent flights correlate in spatial and temporal domains; see Figure 6 (Li, 2018). The along-track registration of the data enables a relatively more straightforward implementation of advanced data processing methods.

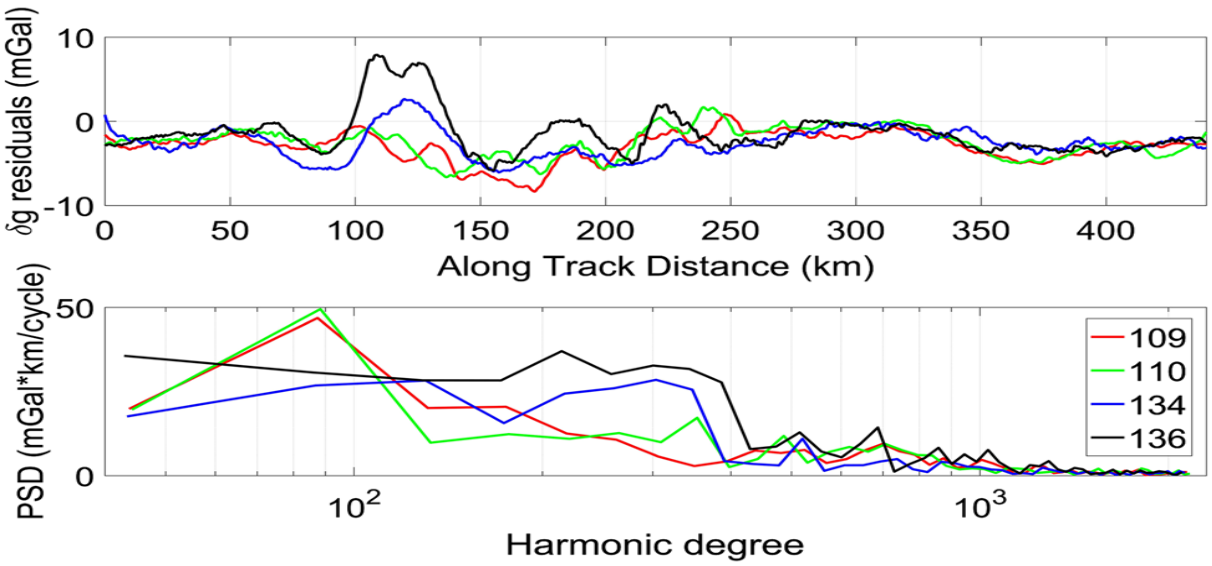

Figure 6. Sampled adjacent flights from typical GRAV-D data sets (Li, 2018).

Li et al. (2016) found that using airborne gravity data improved the geoid model by order of magnitudes in precision compared to multi-mission, multi-year averaged mean lake surfaces in

Lake Michigan; see Figure 7. Combining scalar airborne gravimetry with IMU-based vector gravimetry (Li, 2011) shows promising results to improve further the quality and resolution of the airborne gravity data (Forsberg et al., 2022).

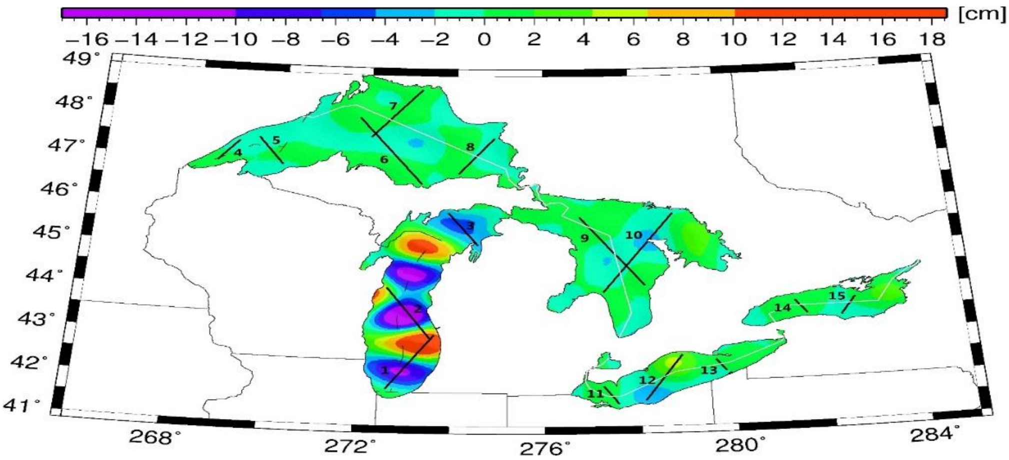

Figure 7. Geoid model improvements due to GRAV-D airborne gravity data in the target area; Li et al. (2016).

Because geoid computation needs topography data to account for the topographic effect, high-quality digital elevation models (DEMs) also play an essential role. NGS tried to generate an accurate high-resolution DEM by combining data from TanDEM-X, MERIT, and USGS 3DEP elevation data sets into xDEM20, which supports xGEOID20. Figure 8 gives a few examples of the improvements, where voids are filled, and artifacts are removed.

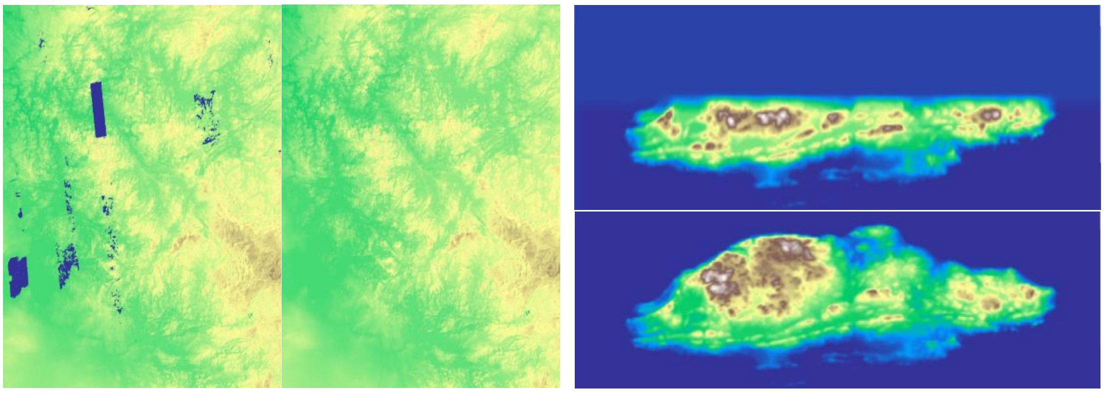

Figure 8. Showcases of the improved in-house DEM (Krcmaric, 2023) (Left panel shows the filled voids; the Right panel shows the fixing of artifacts where half of the island was missing.)

Bathymetry (GEBCO 2021) and ice thickness data (Morlighem et al., 2017) have also been added to the latest version of the DEM. Recently, the DEM is expanded to cover the entire globe.

In addition, the newly released GOCO (Gravity Observation Combination) models (Kvas et al., 2021) are used in defining the long-wavelength content of the geoid model. Satellite altimetry-derived gravity anomalies are used in the open ocean area (Andersen et al., 2018; Sandwell et al., 2013 and 2014).

3. METHODOLOGY UPDATE

Since airborne gravity data plays a vital role in NGS's geoid model modernization, many studies have been conducted to test methods that effectively utilize airborne gravity data in geoid computation. An iterative spherical-harmonic process was developed at NGS to use global spherical harmonics to represent airborne gravity disturbances collected at different altitudes (Holmes 2016). The Geoid Slope Validation Survey of 2011, called GSVS11 (Smith et al., 2013), confirmed that using GRAV-D airborne gravity collected at about 11 km brought the estimated differential geoid accuracy from 1-3cm (without GRAV-D data) down to 1 cm over nearly all distances from 0.4 to 325 km after when combining GRAV-D gravity data; see Figure 9 (Smith et al., 2013).

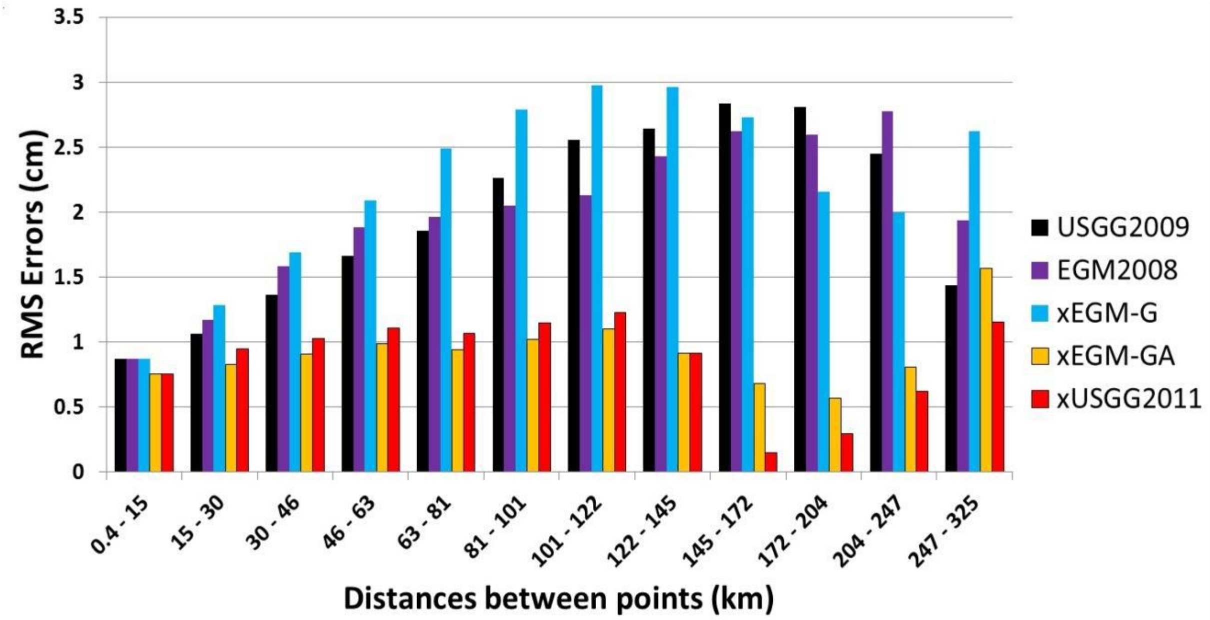

Figure 9. Geoid slope accuracy in various distances. Airborne gravity contributions can be identified by the differences between xEGM-G (cyan) and xEGM-GA (orange) (Smith et al., 2013). (The xEGM-GA blends EGM2008, **G**OCO2s and the **A**irborne gravity data over GSVS 11 area, while the xEGM-G does not contain Airborne gravity in the target area)

Results from two more GSVS surveys (GSVS14 and GSVS17) can be found by Wang et al. (2017) and van Westrum et al. (2021), respectively. Least-squares collocation (LSC), spherical radial basis functions (SRBF) (Li, 2018b), inverse Poisson, and analytical downward continuation methods can be used to fix the spectrum leakage problem when operating global spherical harmonic analysis with airborne gravity data that only covers a part of the globe (Li et al., 2022).

Tesseroid-based mass elements replace prism-based ones to represent better actual topography (Lin and Li, 2022). Three tesseroidal modeling methods based on different combinations of numerical tesseroidal approaches are developed for precise topographic gravity modeling in computing the residual terrain modeling (RTM), terrain correction, and full topographic effects. The developed tools are parallelized using OpenMP to speed up the computation over multiple threads. Moreover, these tools can also include density information rather than assuming a constant value. For example, Figure 10 depicts how to use bathymetry data to model the masses in the lakes for geoid computation (Li et al., 2023). Initial tests in the Great Lakes show consistent geoid model precision improvements across the entire spectrum, i.e., the geoid model precision continuously improves no matter what degree of the Stokes's kernel is used in the integration; see Figure 11 (Li et al., 2022).

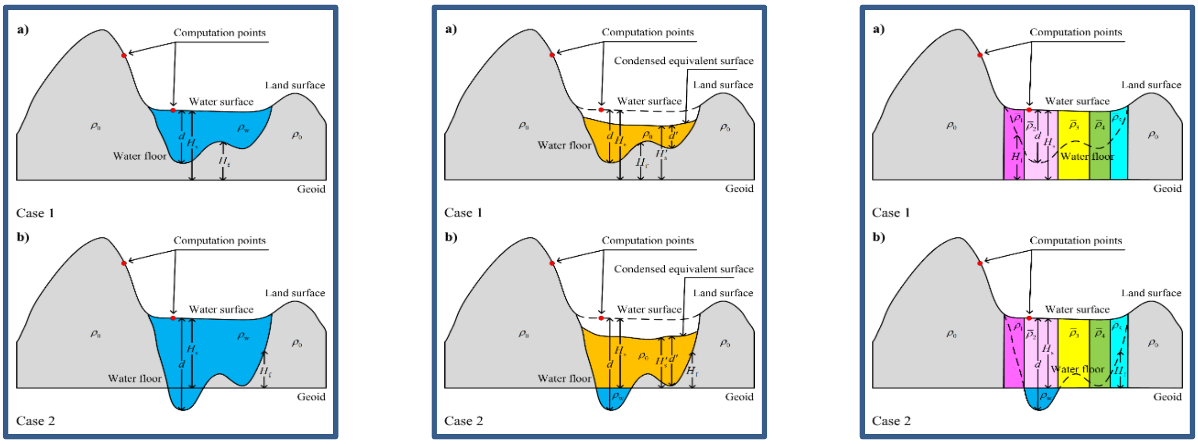

Figure 10. Using bathymetric data to accurately model the water bodies in the lakes for geoid model computation; Li et al., 2023.

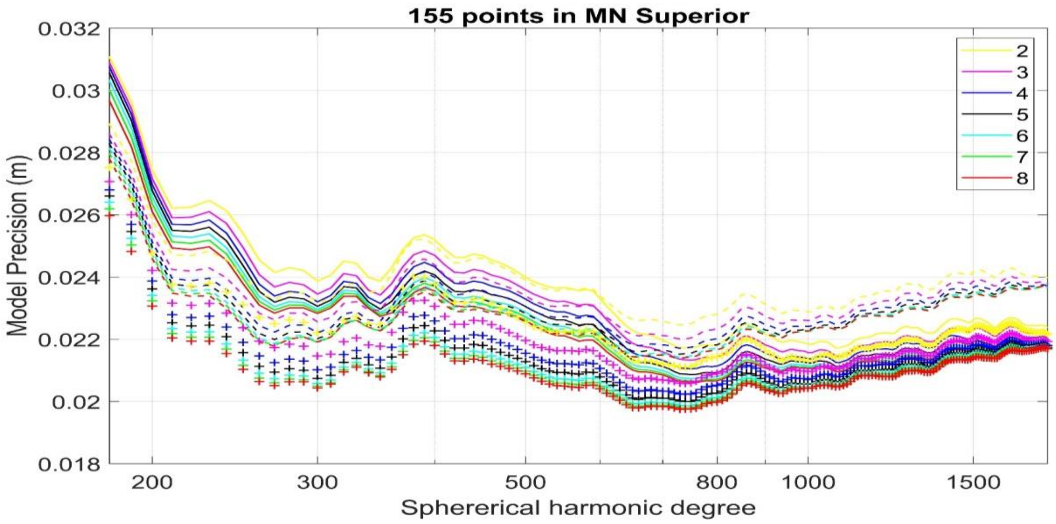

Figure 11. Geoid model precision improvements in Minnesota around Lake Superior, with seven different noise levels (Li et al., 2023).

Harmonic corrections are applied to fix the non-harmonicity problems in valleys below the reference topography used in the RTM approach. Furthermore, ad-hoc algorithms are developed to compute geoid undulation directly on user-defined coordinates rather than interpolating from pre-computed geoid undulation grids, which avoids complicated algorithms (Smith, 2023).

Finally, more accurate geoid quasi-geoid separation terms, the difference between Molodensky's normal height and Helmert's orthometric height (Heiskanen and Moritz, 1967; Smith, 1992 and 1996) are computed from the complete Bouguer correction rather than a simple Bouguer plate correction (Wang et al., 2023). All algorithms are spruced up and closely examined to achieve the most accurate numerical results under the currently available techniques and data sources. Albeit some of these updates only cause mm-level improvements, which probably do not cause any actual changes in many practical applications, they yield scientific completeness at least.

4. SUMMARY

High-quality airborne gravity data are collected by using scalar airborne gravimeters. This generates a uniformly distributed gravity data coverage almost across all US territories with 10-km cross-track resolution and about 30-km along-track resolution after filtering, which may be further improved after combining with vector gravimetry. An in-house high-resolution DEM is generated by integrating third-party models, where artifacts are detected and fixed. Many computational approximations made due to implementation conveniences during geoid modelings, such as prism mass elements, constant densities, and simple Bouguer plates are replaced with more accurate equations. Geoid model changes of up to 40 cm are actual model improvements. Some mm-level model improvements due to algorithm changes may not cause any real impacts but give scientific completeness at least, which can be used to separate other salient errors that still need to be resolved entirely in the literature. Many of the succinct descriptions here involve rather complicated equations and algorithms that can be found in the related topics as cited in the references.

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BIOGRAPHICAL NOTES

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